Solutions Manual to accompany

SOLID STATE RADIO ENGINEERING

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Charles W. Bostian
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SOLUTIONS MANUAL

FOR

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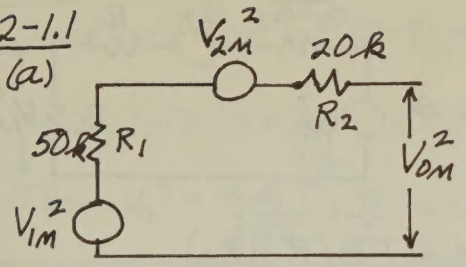
CHAPTER 2
P2-1.1 — 2-1.3

2-1.1

(a)



$V_{OM}^2 = V_{1M}^2 + V_{2M}^2 = 4kTB(R_1+R_2)$

(b) $T = 400K$, $B = 5\times10^6$ Hz, voltmeter has $R_V = \infty$.

(1) $V_{1M}^2 = 4kTBR_1 = 4\times1.38\times10^{-23}\times400\times5\times10^6\times5\times10^4$
$= 5.52\times10^{-9}$

$V_{1M} = \underline{74.3\ \mu V}$

(2) $V_{2M} = \underline{47\ \mu V}$. (3) $V_{OM}^2 = V_{1M}^2 + V_{2M}^2 = 7.73\times10^{-9}$

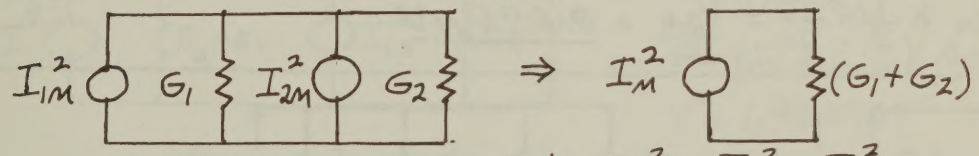
$V_{OM} = \underline{88\ \mu V}$

(c) The voltmeter resistance is $R_V = 100\ k\Omega$. This "loads" the circuit. Voltmeter readings are:

(1) $V_{1MC} = \dfrac{V_{1M}\times100k}{150k} = \underline{49.5\mu V}$

(2) $V_{2MC} = \underline{39.2\ \mu V}$ (3) $V_{OMC} = \underline{51.8\ \mu V}$

2-1.2



The total noise current is $I_M^2 = I_{1M}^2 + I_{2M}^2$

or $I_M = \sqrt{I_{1M}^2 + I_{2M}^2}$

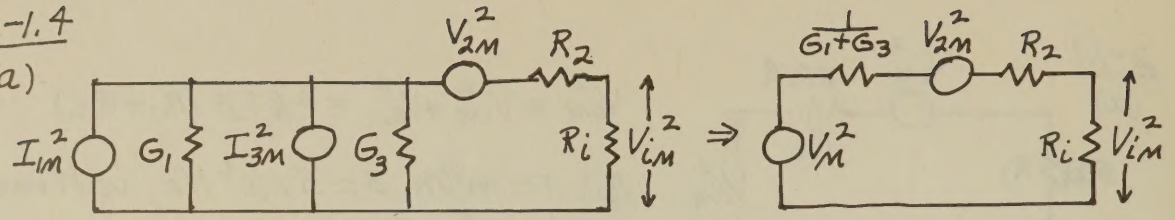
2-1.3 For Fig. 2-1a, the mean-square short-circuit current is $I_M^2 = V_M^2/R^2 = \dfrac{4kTBR}{R^2} = 4kTBG$

where $G = \frac{1}{R}$.

Hence $I_M = (4kTBG)^{1/2}$ as in Fig. 2-1b.

2-1.4

(a)



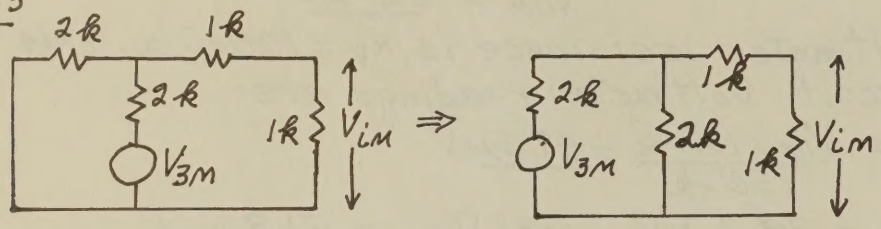
(b) $V_M^2 = \dfrac{I_{1M}^2 + I_{3M}^2}{(G_1+G_3)^2} = \dfrac{4kTB(G_1+G_3)}{(G_1+G_3)^2} = 4kTB(R_1 \| R_3)$

Thevenin resistance $R_T = (R_1 \| R_3) + R_2$

Mean-square Thevenin voltage

$V_{TM}^2 = V_M^2 + V_{2M}^2 = 4kTBR_T$.

2-1.5



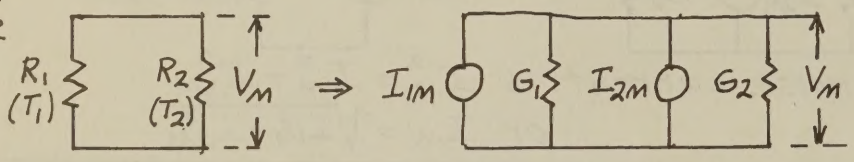
$B = 10^4$, $T = 293K$, $4kT = 1.62 \times 10^{-20}$

$V_{3M}^2 = 4kTBR_3 = 3.24 \times 10^{-13}$; $V_{3M} = 0.569\,\mu V$

$V_{iM} = V_{3M} \times \frac{1}{3} \times \frac{1}{2} = \dfrac{V_{3M}}{6} = \underline{0.095\,\mu V}$

2-1.6

(a)



(b) $I_{1M}^2 = 4kT_1BG_1$; $I_{2M}^2 = 4kT_2BG_2$

$V_M^2 = \dfrac{I_{1M}^2 + I_{2M}^2}{(G_1+G_2)^2} = \dfrac{4kB(T_1G_1 + T_2G_2)}{(G_1+G_2)^2}$

(c) $R_1 = 10k\Omega$, $R_2 = 20k\Omega$, $B = 10^5$, $T_1 = T_2 = 20°C = 293K$

$V_M = \underline{3.28\,\mu V}$

(d) $T_1 = 293K$, $T_2 = 313°C = 586K$

$V_M = \underline{3.80\,\mu V}$

2-1.7

Substitution of (2-9) into (2-8) yields $V_n^2 = \frac{4kT}{2\pi G}\int_0^\infty \frac{d\omega}{1+\left(\frac{\omega}{\omega_2}\right)^2}$.

Substitute $x = \frac{\omega}{\omega_2}$, $dx = \frac{d\omega}{\omega_2}$ to get $V_n^2 = \frac{4kT}{2\pi C}\int_0^\infty \frac{dx}{1+x^2}$

Then $V_n^2 = \frac{4kT}{2\pi C} \times \frac{\pi}{2} = \frac{kT}{C}$

2-3.1 $I^2 = 2qI_{DC}B = 3.2\times10^{-14}$; $I = 0.179\,\mu A$

2-3.2

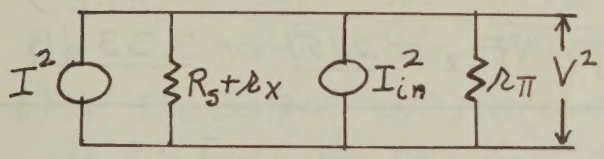
(a) $V_{gn}^2 = 4kTR_sB = 1.6\times10^{-12}$

$V_{bn}^2 = 4kTr_xB = 8\times10^{-14}$

$I_{in}^2 = 2q|I_B'|B = 6.4\times10^{-22}$

$I_{on}^2 = 2q|I_C|B = 3.2\times10^{-19}$

(b) Replace noise sources R_s and r_x by their Norton equivalent.



$I^2 = \frac{V_{gn}^2 + V_{bn}^2}{(R_s+r_x)^2} = 1.524\times10^{-20}$

$V^2 = (I^2 + I_{in}^2)\left[r_\pi \| (R_s+r_x)\right]^2$

$V^2 = 1.46\times10^{-12}$

(c) Let $I_{cn}^2 = I_{on}^2 + g_m^2V^2 = 3.2\times10^{-19} + (4\times10^{-3})^2 \times 1.46\times10^{-12}$

$I_{cn}^2 = 23.68\times10^{-18}$

(d) $V_{no}^2 = I_{cn}^2R_L^2 = 5.92\times10^{-8}$

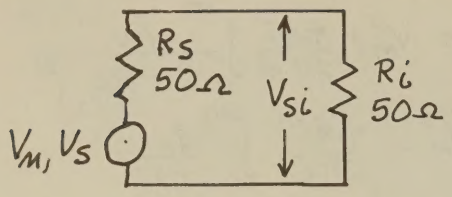
(e) $V_{R_L}^2 = 4kTBR_L = 8\times10^{-12}$ (negligible compared to V_{no}^2.)

(f) V_{on}^2 due to V_{gn}^2:

$V_{on}^2 = V_{gn}^2 \times \left(\frac{r_\pi}{R_g+r_x+r_\pi}\right)^2 g_m^2R_L^2 = 5.33\times10^{-8}$

(g) $NF = \frac{V_{no}^2}{V_{on}^2} = 1.11$ or $0.45\,dB$

2-4.1



For $NF_{dB} = 3$, $NF = 2$

(a) $T_e = T_0(NF-1) = 290(2-1) = \underline{290\ K}$

(b) $V_n^2 = 4kTBR_s = 2.4 \times 10^{-13}$

At the amplifier input,

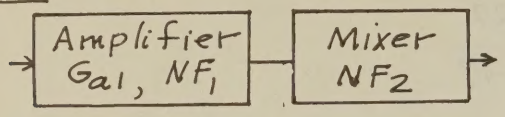
$V_{ni}^2 = \frac{V_n^2}{4} = 6 \times 10^{-14}$

$V_{si}^2 = 10^{-12}$

Input: $SNR_i = \frac{V_{si}^2}{V_{ni}^2} = \underline{16.67}$ or 12.2 dB

Output: $SNR_o = \frac{SNR_i}{NF} = \underline{8.33}$ or 9.2 dB

2-5.1



NF_1 (dB) = 3, $NF_1 = 2$

G_{a1} (dB) = 5.5, $G_{a1} = 3.55$

NF_2 (dB) = 4, $NF_2 = 2.51$

$NF_{1,2} = NF_1 + \frac{NF_2 - 1}{G_{a1}} = \underline{2.425}$ or $\underline{3.85\ dB}$

2-5.2

G_{a1} (dB) = 10, $G_{a1} = 10$; $NF_{1,2} = \underline{2.151}$ or $\underline{3.33\ dB}$

2-5.3

NF_2 (dB) = 8, $NF_2 = 6.31$

With $G_{a1} = 3.55$, $NF_{1,2} = \underline{3.5}$ or $\underline{5.43\ dB}$

With $G_{a1} = 10$, $NF_{1,2} = \underline{2.53}$ or $\underline{4.03\ dB}$

2-5.4

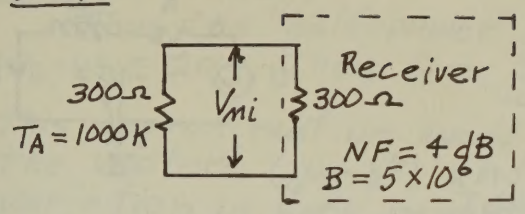
NF_1 (dB) = 2.5, $NF_1 = 1.78$

NF_2 (dB) = 8, $NF_2 = 6.3$

$NF_{1,2}$ (dB) = 3.5, $NF_{1,2} = 2.24$

$G_1 = \frac{NF_2 - 1}{NF_{1,2} - NF_1} = \underline{11.52}$ or $\underline{10.61\ dB}$

2-5.5



(a) $V_{ni}^2 = \frac{4kT_A BR}{4} = 2.07\times10^{-11}$

$V_{ni} = \underline{4.55\,\mu V}$

(b) Input SNR of 30dB gives

$SNR_i = 1000 = \frac{V_{si}^2}{V_{ni}^2}$

$V_{si} = \underline{144\,\mu V}$

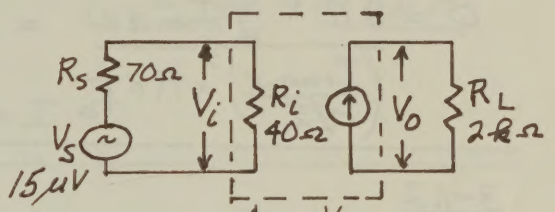
(c) For $NF_{dB} = 4$, $NF = 2.51$

From (2-58), $NF_{act} = 1 + (NF-1)\frac{T_o}{T_s} = 1 + (2.51-1)\frac{290}{1000} = \underline{1.438}$

2-5.6

Since no output impedance was specified, it will be assumed very large compared to R_L; i.e., a current source.

Thermal noise produced by Rs is given by $V_{ns}^2 = 4kTBR_s = 8.96\times10^{-12}$



$A_V = \frac{V_o}{V_i} = 50$

T = 290 K

$B = 8\times10^6$

(a) At the input,

$SNR_i = \frac{V_{si}^2}{V_{ni}^2} = \frac{V_s^2}{V_{ns}^2} = \frac{(15\times10^{-6})^2}{8.96\times10^{-12}} = \underline{25.1}$ or $\underline{14\,dB}$

(b) At the output, all thermal noise generated in R_L appears as part of V_o. Let $V_{nL}^2 = 4kTBR_L = 2.56\times10^{-10}$

The total mean-square noise output is $(V_{nL}^2 + A_V^2 V_{ni}^2)$

$A_V^2 V_{ni}^2 = 8.96\times10^{-12}\times\left(\frac{40}{110}\right)^2\times2500 = 2.962\times10^{-9}$

The signal output is $V_{so}^2 = V_s^2\times\left(\frac{40}{110}\right)^2\times2500 = 7.438\times10^{-8}$

Then $SNR_o = \frac{V_{so}^2}{V_{nL}^2 + V_{ni}^2 A_V^2} = \frac{7.438\times10^{-8}}{2.56\times10^{-10} + 2.96\times10^{-9}} = \underline{23.1}$

or $\underline{13.64\,dB}$

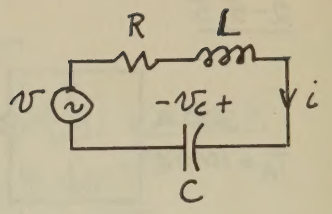
(C) Amplifier has $NF = 2 = \frac{SNR_i}{SNR_o}$, and $SNR_o = \frac{25.1}{2}$ if the noise due to R_L is not considered.

Let the mean-square output noise voltage be V_{no}^2

$V_{no}^2 = 2\times V_{ni}^2 A_V^2 = 5.93\times10^{-9}$

The true $SNR_o = \frac{V_{so}^2}{V_{no}^2 + V_{nL}^2} = \underline{12.03}$ or $\underline{10.8\,dB}$

CHAPTER 3
P3-1.1 — 3-1.4



3-1.1

At resonance, $\omega_o^2 = \frac{1}{LC}$

Let $v = V\cos\omega_o t$

$$i = \frac{V}{R}\cos\omega_o t = I_{max}\cos\omega_o t$$

$$v_C = \frac{I_{max}}{\omega_o C}\sin\omega_o t$$

Stored energy in L and C is: $w_L = \frac{1}{2}Li^2$, $w_C = \frac{1}{2}Cv_C^2$

Combine and show that $w_L + w_C = \frac{L}{2}I_{max}^2\underbrace{(\cos^2\omega_o t + \sin^2\omega_o t)}_{=1}$

Substitute into (3-4):

$$Q = \frac{2\pi \times \frac{L}{2}I_{max}^2}{\left(\frac{I_{max}}{\sqrt{2}}\right)^2 \times R \times \frac{1}{f_o}} = \frac{\omega_o L}{R}.$$

3-1.2

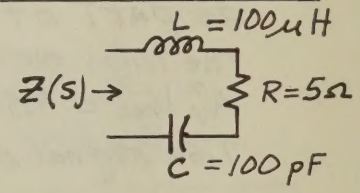
$$|V_C| = \frac{I_{max}}{\omega_o C} = \frac{V}{R\omega_o C} = \frac{V\omega_o L}{R} = QV$$

3-1.3

$L = 100\,\mu H = 10^{-4}$, $C = 100\,pF = 10^{-10}$, $R = 5$

(a) $f_o = 1.59\,MHz$, $\omega_o = 10^7$; (b) $Q = 200$; (c) $V_C = 2\,mV$

3-1.4



$Z(s)$ has a pole at $s=0$ and zeros at

$\gamma = \alpha + j\beta = -2.5\times10^4 + j10^7$ and

$\gamma^* = \alpha - j\beta = -2.5\times10^4 - j10^7$

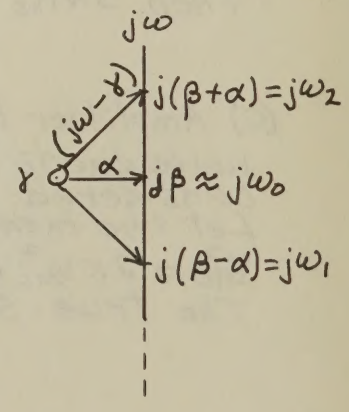
For $s = j\omega$, $Z(j\omega) = \frac{L(j\omega-\gamma)(j\omega-\gamma^*)}{j\omega}$

Since $\beta = 400\alpha$, the variation in $Z(j\omega)$ near the zero at γ is caused almost entirely by the variation in the vector $(j\omega-\gamma)$. (The ratio $\frac{j\omega-\gamma^*}{j\omega}$ remains nearly constant). The s plane in the vicinity of γ is shown here. At

$\omega = \beta - \alpha$, $j\omega - \gamma = \sqrt{2}\,\alpha\,\angle -45°$

$\omega = \beta$, $j\omega - \gamma = \alpha\,\angle 0°$

$\omega = \beta + \alpha$, $j\omega - \gamma = \sqrt{2}\,\alpha\,\angle +45°$



(continued on next page)

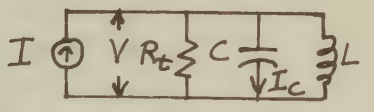
3-1.4 (continued)

Thus the "half-power" bandwidth for $Z(j\omega)$ is approximately $\omega_2-\omega_1 = 2\alpha$. Then $Q \approx \frac{\omega_0}{\omega_2-\omega_1} = \frac{\beta}{2\alpha}$.

This approximation would not hold for $\beta = 3\alpha$ because the vectors $(j\omega-\delta^*)$ and $(j\omega)$ would exhibit appreciable variation in both magnitude and angle.

3-2.1

$R_t = 2000\,\Omega$, $f_0 = 10^7$ Hz, $B = 250$ kHz



(a) From (3-12), $Q_t = \frac{f_0}{B} = \underline{40}$

$C = \frac{1}{2\pi B R_t} = \underline{318\text{ pF}}$ $\quad L = \frac{1}{\omega_0^2 C} = \underline{0.8\,\mu\text{H}}$

(b) $I = 2$ mA, $V = I \times R_t = 4$V

$I_C = V \times \omega_0 C = \underline{80\text{ mA}} = Q \times I$

3-3.1

$R_t = 2$ kΩ, $R_L = 5\,\Omega$, $C_0 = 20$ pF, $f_0 = 1.6$ MHz, $\omega_0 \approx 10^7$ rad/s.

(a) Let $r_c = 0$ and $C_0 + C = C_T$.

From $\frac{R_t}{R_L} = Q_t^2 + 1$, $Q_t = 19.98$

$L = \frac{Q_t R_L}{\omega_0} = \underline{10\,\mu\text{H}}$; $C_T = \frac{Q_t}{\omega_0 R_t} = \underline{1000\text{ pF}}$; $C = \underline{980\text{ pF}}$

$B = \frac{f_0}{Q_t} = \underline{80.1\text{ kHz}}$.

(b) If $Q_L = 50 = \frac{\omega_0 L}{r_c}$; $\underline{r_c = 2\,\Omega}$. Then $R_t = \frac{L}{C_T(R_L + r_c)} = \underline{1429\,\Omega}$

(c) $r_c = \frac{\omega_0 L}{Q_L} = 2\times10^5 L$.

Let $r_c = kL$, where $k = 2\times10^5$. Define $R_S = R_L + r_c = R_L + kL$.

Then $Q_T = \frac{\omega_0 L}{R_S} = \frac{\omega_0 L}{R_L + kL}$

$$R_t = R_S(Q_t^2+1) = \frac{\omega_0^2 L^2 + R_S^2}{R_S} = \frac{\omega_0^2 L^2 + (R_L+kL)^2}{R_L + kL}$$

From this, $L^2 - \frac{kL(R_t - 2R_L)}{\omega_0^2 + k^2} - \frac{R_L(R_t - R_L)}{\omega_0^2 + k^2} = 0$.

Solve for L to get $\underline{L = 12.18\,\mu\text{H}}$; $R_S = 5 + kL = 7.436\,\Omega$

$C_T = \frac{L}{R_t R_S} = \underline{819\text{ pF}}$ $\quad r_c = 2.436\,\Omega$

$C = \underline{799\text{ pF}}$

3-3.2

Use Table 3-3.2. $R_t = 4\,k\Omega$, $R_1 = 5\,\Omega$, $f_o = 20\,MHz$.

$Q_t^2 \approx R_t/R_1 = 800$. $\underline{Q_t = 28.28}$

$L = \frac{Q_t R_1}{\omega_o} = \underline{1.125\,\mu H}$. $C = \frac{1}{\omega_o^2 L} = \underline{56.3\,pF}$

$B = f_o/Q_t = 707\,kHz$.

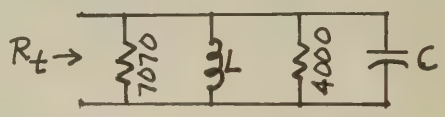
3-3.3

$L = 1.125\,\mu H$, $R_1 = 5\,\Omega$, $C = 56.3\,pF$, $f_o = 20\,MHz$, $Q_L = 50$

Coil resistance $r_c = \frac{\omega_o L}{Q_L} = \underline{2.827\,\Omega}$.

R_1 in series with C transforms to 4000 Ω in parallel.

r_c in series with L transforms to the parallel equivalent $R_p = r_c(Q_L^2+1) = 7070\,\Omega$.



$\underline{R_t} = 4000 \| 7070 = \underline{2555\,\Omega}$

$\underline{Q_t} = \omega_o C R_t = \frac{R_t}{\omega_o L} = \underline{18.1}$; $\underline{B = f_o/Q_t = 1.11\,MHz}$

3-3.4

If the source resistance $R_s = 6\,k\Omega$ is added in parallel with $4\,k\Omega$ (the R_t of the parallel circuit), the new R_t becomes $R_t' = 6000 \| 4000 = \underline{2400\,\Omega}$

$Q_t' = R_t'/\omega_o L = \underline{16.98}$; $B = f_o/Q_t' = \underline{1.18\,MHz}$.

3-3.5 The easy route to this derivation starts with

$$Y = \frac{1}{j\omega L} + \frac{1}{R + \frac{1}{j\omega C}} = \frac{\omega^2 C^2 R}{1+\omega^2 C^2 R^2} + j\left[\frac{\omega C}{1+\omega^2 C^2 R^2} - \frac{1}{\omega L}\right]$$

At resonance the imaginary part is equal to zero, giving the expression for ω_o. $\left[\omega_o^2 = \frac{1}{LC - R^2C^2}\right]$

The real part of Y then gives $Y_t = \frac{CR}{L}$ or $R_t = \frac{L}{RC}$.

3-6.1 Use Table 3-6.1

$Q_t = 100$, $Q_p \approx 63.25$, $C = \frac{1}{2\pi B R_t} = 3183\,pF$

$\underline{L = 0.318\,\mu H}$

$\underline{C_1 = 8671\,pF}$ $\underline{C_2 = 5029\,pF}$

3-6.2

$f_o = 5\times10^6$ Hz, $\omega_o = 10^7\pi$ rad/s, $B = 5\times10^4$ Hz, $Q_t = f_o/B = 100$.
Choose $R_t = 10\,k\Omega$. Then $C = \frac{1}{2\pi B R_t} = 318.3\,pF = \frac{C_1 C_{se}}{C_1 + C_{se}}$
and $L = \frac{1}{\omega_o^2 C} = 3.183\,\mu H$

For the tapped-capacitor side use Table 3-6.1.
$N_C^2 = R_t/R_2 = 25$; $N_C = 5$; $Q_p \approx Q_t/N_C = 20$
$C_2 = N_C C = 1591\,pF$. $C_1 = C_2/(N_C - 1) = 398\,pF$

For the tapped inductor use Table 3-6.2.
$N_L^2 = R_t/R_1 = 10$, $N_L = 3.162$; $Q_p = Q_t/N_L = 31.62$
$L_2 = L/N_L = 1\,\mu H$; $L_1 = L - L_2 = 2.183\,\mu H$

Note: If the circuit is driven by a source with resistance $R_s = R_1 = 1\,k\Omega$, the transfer function will exhibit a bandwidth $B = 100$ kHz and overall Q of 50.

3-7.1

(a) $L_{13} = L_{12} + L_{23} + 2M$; $9 = 4 + 2 + 2M$; $M = 1.5\,\mu H$
$M = k\sqrt{L_{12}L_{23}}$; $k = \frac{1.5}{\sqrt{8}} = 0.53$

(b) $V_{12} = V_{13} \times \frac{j\omega(L_{12}+M)}{j\omega L_{13}} = \frac{5.5}{9}\times 10 = 6.1\,V$

$V_{23} = V_{13} \times \frac{(L_{23}+M)}{L_{13}} = 3.9\,V$

3-7.2 Circuit of Fig. 3-16 with $R_t = 250\,\Omega$, $R_2 = 10\,\Omega$, $k = 0.25$, $L = 2\,\mu H$, $f_o = 4$ MHz.

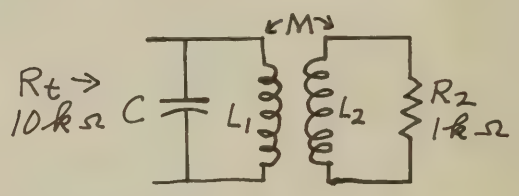
$R_t/R_2 = 25$, $\omega_o L/R_2 = 5.03$
From Fig. 3-18, $1/N \approx 0.25$, $N = 4$
From Fig. 3-22, $D \approx 1.5$ [As a check, use (3-45) to get $R_t/R_2 = N^2 D = 16\times1.5 = 24$.]
From Fig. 3-23, $E \approx 1.05$
$-B_i = \omega_o C = \frac{E}{\omega_o L} = 0.0209\,\mho$
$C = \frac{0.0209}{\omega_o} = 831\,pF$
The tap should be 25% of the way from the bottom to the top of the coil.

3-7.3

$R_2 = 50$, $R_t = 200$, $L = 2\,\mu H$, $k = 0.1$, $f_0 = 30$ MHz.
$R_t/R_2 = 4$, $\omega_0 L/R_2 = 7.54$, and $\omega_0 L = 377\,\Omega$.
From Fig. 3-17, $1/N \approx 0.75$, $N = 1.333$.
The tap should be at 75% of the total turns.
From Fig. 3-20, $D \approx 2.25$. [$N^2 D = 4$ as it should.]
From Fig. 3-21, $E \approx 3$.
$-B_i = \omega_0 C = \frac{E}{\omega_0 L} = \frac{3}{377} = 7.96$ mU. $C = \frac{7.96 \times 10^{-3}}{\omega_0} = 42.2$ pF

3-8.1

Use Table 3-8.
$f_0 = 16$ MHz, $B = 160$ kHz
$Q_t = f_0/B = 100$
$C = \frac{1}{2\pi B R_t} = 99.5$ pF
$L_t = \frac{1}{\omega_0^2 C} = 0.995\,\mu H$



$N = \sqrt{\frac{R_t}{R_2}} = \sqrt{10} = 3.162$

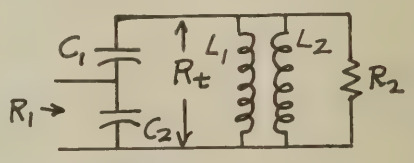
(a) With $Q_p = 1$; $k = 0.141$, $L_2 = 10\,\mu H$, $L_1 = 1.005\,\mu H$, $M = 0.446\,\mu H$.

(b) With $Q_p = 10$; $k = 0.316$, $L_2 = 1\,\mu H$, $L_1 = 0.995\,\mu H$, $M = 0.315\,\mu H$.

3-8.2

Use the circuit shown above, with $R_t = 1000\,\Omega$, $R_2 = 400\,\Omega$, $f_0 = 5 \times 10^6$ Hz, $\omega_0 = 10^7\pi$, $B = 5 \times 10^4$ Hz.
From P3-6.1, $Q_t = 100$, $C = 3183$ pF, $L_t = 0.3183\,\mu H$.
From Fig. 3-17, $k_{min} = 0.14$ and $Q_{p,min} \approx 1$.
Use Table 3-8. For $Q_p = 5$, $k = 0.228$, $L_2 = 2.55\,\mu H$, $L_1 = 0.319\,\mu H$, $M = 0.206\,\mu H$.
For $Q_p = 10$, $k = 0.317$, $L_2 = 1.27\,\mu H$, $L_1 = 0.319\,\mu H$, $M = 0.202\,\mu H$.

Note! If a tapped-capacitor circuit is used as shown here and $R_2 = 400\,\Omega$ is transformed to $R_t = 10$ kΩ and back to $R_1 = 1$ kΩ the circuit values are!



For $Q_p = 10$, $k = 0.317$, $C = 318.3$ pF, $L_t = 3.183\,\mu H$, $\approx L_1$, $L_2 = 1.27\,\mu H$, $M = 1\,\mu H$, $C_1 = 398$ pF, $C_2 = 1591$ pF.

3-8.3 (a)

$f_T = 100 \times 3 = \underline{300\,MHz}$, $\omega_T = 1.885 \times 10^9$ rad/s. at $I_C = 10\,mA$.
Assume ω_T is not a function of I_C since no data is available for computing the value at $I_C = 1\,mA$.
At $I_C = 10\,mA$, $g_m = 40 I_C = \underline{0.4\,\mho}$ and $C_\pi = \frac{g_m}{\omega_T} - C_\mu = \underline{208\,pF}$
At $I_C = 1\,mA$, $\underline{g_m = 0.04\,\mho}$ and $\underline{C_\pi = 17.2\,pF}$.

(b) Node analysis of Fig. P3-8.3b gives:

$$A_V = \frac{V_2}{V_1} = \frac{-g_m + j\omega C_\mu}{g_L + j\omega C_\mu}, \quad \text{where } g_L = \frac{1}{R_L} \qquad (1)$$

$$Y_i = G_i + jB_i = G_i + j\omega C_i \qquad (2)$$

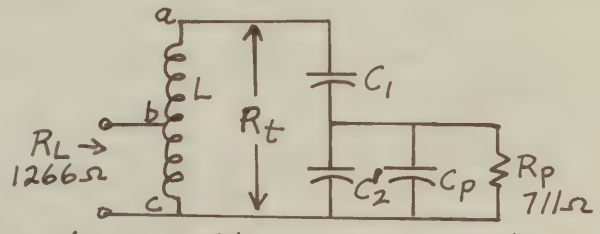
$$G_i = \frac{1}{R_p} = g_\pi + \frac{\omega^2 C_\mu^2 R_L (1 + g_m R_L)}{1 + \omega^2 C_\mu^2 R_L^2} \qquad (3)$$

$$C_i = C_p = \frac{B_i}{\omega} = C_\pi + C_\mu \left[\frac{1 + g_m R_L}{1 + \omega^2 C_\mu^2 R_L^2}\right] \qquad (4)$$

Use (1) with $|A_V| = 50$ to solve for $\underline{R_L = 1266\,\Omega}$
Then $G_i = 1.407 \times 10^{-3}\,\mho$, $\underline{R_p = 711\,\Omega}$

(c) $\underline{C_i = C_p = 218\,pF}$

(d) The network transforms $711\,\Omega$ to $1266\,\Omega$ with $f_o = 5\,MHz$, $B = 50\,kHz$, and $Q_t = 100$. The coil inductance is chosen arbitrarily at $\underline{L = 2\,\mu H}$. The coil has $Q_L = 200$, hence its equivalent parallel resistance is $\underline{R_{t1} = \omega_o L Q_L = 12566\,\Omega}$.



Capacitors C_2 ($= C_2' + C_p$) and C_1 must transform R_p to an equivalent parallel resistance $\underline{R_{t2} = 12566\,\Omega}$.
Then $R_t = R_{t1} + R_{t2} = 6283\,\Omega$ gives the desired $Q_t = 100$.
$C = \frac{1}{\omega_o^2 L} = \frac{Q_t}{\omega_o R_t} = 507\,pF$. Use Table 3-6.1 to find
$N = 4.204$, $Q_p = 47.6$, $C_2 = NC = 2131\,pF$, $\underline{C_2' = 2131 - 218 = 1913\,pF}$
$\underline{C_1 = C_2(N-1) = 665\,pF}$

(e) Refer to Sec. 3-7. Assume $k = 0.1$ for the coil. $\frac{\omega_o L}{R_L} = 0.05$.
Hence the coil behaves like a perfect transformer.
For total turns "n", the number of turns from b to c is
$n \times \sqrt{R_L/R_t} = \underline{0.45\,n}$

CHAPTER 4

P4-2.1 — 4-5.2

Note: Computer programs were used for some of the calculations in these problems. Hence only the final answers are available.

4-2.1 (a)

$f_T = 4 \times 250 = 1000$ MHz. $g_m = \frac{4 \times 10^{-3}}{26} = 153.8$ m℧

$r_\pi = \frac{h_{fe}}{g_m} = 650\ \Omega$. $g_\pi = 1/r_\pi = 1.538$ m℧

$r_x = h_{ie} - r_\pi = 10\ \Omega$. $g_x = 100$ m℧

$C_\mu = C_{ob} = 2$ pF. $C_\pi = \frac{g_m}{\omega_T} - C_\mu = 22.5$ pF

Assume $r_{ce} = 50\ k\Omega$. Then $g_o = \frac{1}{r_{ce}} = 0.02$ m℧

(b) Use the formulas in Appendix 4-2, with $f = 10$ MHz.

$Y = 101.5 + j\,1.54$ m℧

$y_{ie} = 1.52 + j\,1.52$ m℧ $y_{fe} = 152 + j\,0$ m℧

$y_{re} = -j\,0.124$ m℧ $y_{oe} = 0.0175 + j\,0.316$ m℧

4-3.1

From (4-15), $C = 2.57$

4-3.2

From (4-16), $G_L = 1.657$ m℧

4-3.3

(a) From (4-16), $K = 4.08$ (b) $K = 0.82$

4-4.1 Use (4-17) and (4-18)

(a) $y_{ic} = 2.7 + j\,6.3$ m℧ $y_{fc} = 53 - j\,21.5$ m℧

$y_{oc} = 0.1 + j\,1.0$ m℧ $y_{rc} = 0$

$C = 0$ and K is infinite

(b) $C = -2.6$ (unstable) $K = 1.25$

4-5.1

From (4-25), MAG = 22,533

4-5.2

From (4-24), $G_T = 39$ or 15.9 dB

From (4-20), $G_p = 210$ or 23.2 dB

4-7.1

(a) $y_{ic} = 3.6 + j17.4$ m℧ $\quad y_{fc} = 29 - j9.4$ m℧
$y_{oc} = 0.0029 + j0$ " $\quad y_{rc} = -0.04 + j0$ "

(b) From (4-15), $C = 1.03$
" (4-25), $MAG = 22{,}250$
" (4-16), $G_L = 0.01437$ m℧, $R_L = 69.6$ kΩ.

4-7.2

$G_S = 9.98$ m℧, $\quad G_L = 0.00804$ m℧
$B_S = 18.34$ m℧, $\quad B_L = 0.0319$ m℧ by iteration
$G_T = 1301$

4-7.3

From (4-16), $K = 4.01$
By iteration, $B_S = -27.07$ m℧
$B_L = -1.707$ "
From (4-24), $G_T = 111$ or 20.46 dB

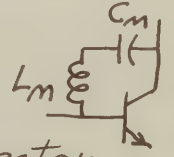
4-10.1

(a) Given $I_C = 1$ mA and $R_E = 3$ kΩ, $V_E = 3$ V
$V_{CC} = V_E + V_{CE} = 13$ V

Transistor data sheet gives $h_{FE} = 25$; $\therefore I_B = \frac{1}{25} = 0.04$ mA
Assume $V_{BE} = 0.7$ V and $R_B = R_1 \| R_2 = 30$ kΩ
$V_{BB} = V_{BE} + I_B[R_B + R_E(h_{FE}+1)] = 5.02$ V
$\underline{R_1 = \frac{R_B V_{CC}}{V_{BB}} = 78\ k\Omega}$. $\quad \underline{R_2 = \frac{R_1 R_B}{R_1 - R_B} = 49\ k\Omega}$

C_1, C_2, and C_E can be on the order of 0.01–0.05 μF if their self-resonance frequency is > 5 MHz. (See Appendix 4-5.)

(b) $\underline{y_x = y_{rt} = 0 - j0.6\ m℧}$.



This represents an inductive reactance $\omega_o L_M = \frac{10^3}{0.6} = 1667\ \Omega$ connected from collector to base. A DC block (C_M) is also required.
If L_M is adjustable, use $L_M = 53$ μH, $C_M = 0.01$ μF; or use fixed $L_M = 100$ μH and adjust by varying C_M (10–30 pF trimmer capacitor)

4-10.1 (continued)

(b) From (4-17) and (4-18) the composite parameters are:

$y_{ic} = 3.6 + j17.4$ m℧ $\quad y_{fc} = 29 - j9.4$ m℧

$y_{oc} = 0.0029 + j0$ " $\quad y_{rc} = 0$

(c) With $y_{rc} = 0$, the circuit is unilateralized.

$Y_{1u} = y_{ic} = 3.6 + j17.4$ m℧; $Y_{2u} = y_{oc} = 0.0029 + j0$ m℧ (no output capacitance).

$C_t = \frac{1}{\omega_0^2 L_t} = 101.3\,pF$

At the input, $B_{1u} = 17.4$ m℧ $= \omega_0 C_i$; $C_i = 554\,pF$.

Resonate with a shunt inductor, $L_i = \frac{1}{\omega_0 B_{1u}} = 1.83\,\mu H$ connected to ground from the left side of C_1.

(d) If $R_g = 200\,\Omega$, $G_g = 5$ m℧ and the Q_i on the input side is $Q_i \approx \frac{\omega_0 C_i}{G_g + G_{1u}} = \frac{0.0174}{0.0086} = 2.02$

On the output side, $Q_o \approx \frac{\omega_0 C_t}{G_L + G_{2u}} = \frac{3.18}{0.5029} = 6.32$

Bandwidth is limited by Q_o.

$B \approx f_o/Q_o = 790$ kHz.

(e) From (4-9), $A_V = \frac{V_o}{V_i} = \frac{-y_{fc}}{y_{oc} + Y_L} = 60.62 \angle 162°$

$\frac{V_i}{V_s} = \frac{R_i}{R_i + R_g} = 0.582$ where $R_g = 200\,\Omega$, $R_i = \frac{1}{G_{1u}} = 278\,\Omega$

$\frac{V_o}{V_s} = 35.26 \angle 162°$

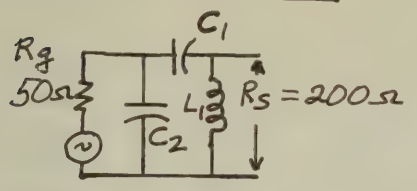
(f) From (4-23), $G_{Tu} = \frac{P_o}{P_{as}} = \frac{V_o^2/R_L}{V_s^2/4R_g} = \left|\frac{V_o}{V_s}\right|^2 \times \frac{800}{2000} = 497$

(g) With $V_s = 1$ mV, $P_o = 0.62\,\mu W$

4-10.2

$f_o = 5\,MHz$, $B = 250\,kHz$, $\therefore Q_t = \frac{5}{0.25} = 20.$

Input Network



$Q_i = 20 = \frac{\omega_o C_{ti}}{G_s + G_{1u}} = \frac{\omega_o C_{ti}}{G_{ti}}$

From the above [or (4-46)]

$C_{ti} = 5475\,pF.$

From P4-10.1, C_i (of transistor) = 534 pF

Thus, for the matching network, $C = C_{ti} - C_i = \underline{4921\,pF}$

Use Table 3-6.1. From (4-50) the network has

$Q_t = \frac{\omega_o C}{G_s} = 30.92$

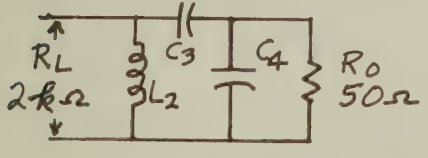
$N^2 = \frac{R_s}{R_g} = 4,\ N = 2,\ Q_p \approx \frac{Q_t}{N} = 15.46$

Approximate formulas give $\underline{C_2} = NC = \underline{9842\,pF}$

$\underline{C_1} = \frac{C_2}{N-1} = \underline{9842\,pF}$

also $\underline{L_1} = \frac{1}{\omega_o^2 C_{ti}} = \underline{0.185\,\mu H}$

Output Network



$G_L = \frac{1}{2000} = 0.5\,m\mho$

$G_{to} = G_L + G_{2u} = 0.5029\,m\mho$

$C_{to} = \frac{G_{to}}{2\pi B} = 320\,pF$

$\underline{L_2} = \frac{1}{\omega_o^2 C_{to}} = \underline{3.16\,\mu H}$

$B_{2u} = 0$, hence all of C_{to} is provided by the matching network.

$N^2 = \frac{2000}{50} = 40;\ N = 6.325.$

Network $Q_t = \frac{\omega_o C_{to}}{G_L} = 20.1;\quad Q_p \approx \frac{Q_t}{N} = 3.18$ hence exact formulas are used.

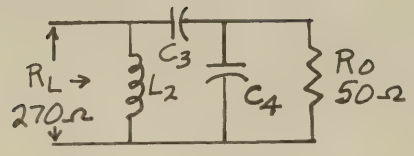
$Q_p = \left[\frac{Q_t^2+1}{N^2} - 1\right]^{1/2} = 3.02$

$\underline{C_4} = \frac{Q_p}{\omega_o R_o} = \underline{1923\,pF}$

$C_{se} = C_4\left(\frac{Q_p^2+1}{Q_p^2}\right) = 2134\,pF$

$\underline{C_3} = \frac{C_{se} C_{to}}{C_{se} - C_{to}} = \underline{376\,pF}$

4-10.3



Output Network design for Example 4-10.1

$G_L = \frac{1}{R_L} = 3.6975\ m\mho$

$G_2 = 0.4714\ m\mho$

$G_{to} = G_L + G_2 = 3.226\ m\mho$

Transistor output capacitance $C_o = \frac{B_2}{\omega_0} = \frac{3.07\times10^{-3}}{\omega_0} = 98\,pF$

For $B = 0.25\,MHz$, $C_{to} = \frac{G_{to}}{2\pi B} = 2054\,pF$

The matching network must supply $C = C_{to} - C_o = 1956\,pF$

The network has $Q_t = \frac{\omega_0 C}{G_L} = 16.62$

Use Table 3-6.1:

$N^2 = \frac{R_L}{R_0} = 5.4$; $N = 2.324$; $Q_p \approx \frac{Q_t}{N} = 7.15$

Use exact formulas.

$$Q_p = \left[\frac{Q_t^2+1}{N^2} - 1\right]^{1/2} = 7.08$$

$$\underline{C_4} = \frac{Q_p}{\omega_0 R_0} = \underline{4507\,pF}$$

$$C_{se} = C_4\left(\frac{Q_p^2+1}{Q_p^2}\right) = 4597\,pF$$

$$\underline{C_3} = \frac{C_{se}C}{C_{se}-C} = \underline{3405\,pF}$$

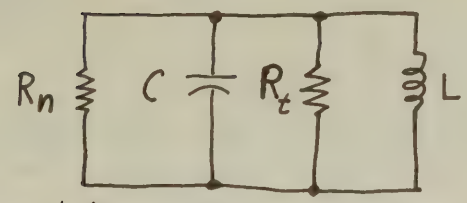
$$\underline{L_2} = \frac{1}{\omega_0^2 C} = \underline{0.493\,\mu H}$$

CHAPTER 5
P5-2.1 — 5-5.1

5-2.1

(a)

Equivalent Circuit:



Oscillation requires $R_t > R_n$

R_t is the equivalent parallel resistance of the inductor. In this problem we are given the series equivalent values:

$$R_t = \frac{R_s^2 + (\omega L_s)^2}{R_s}$$

Hence $R_t > |R_n|$ means $\dfrac{R_s^2 + (\omega L_s)^2}{R_s} > |R_n|$

or $\omega > \sqrt{\dfrac{|R_n| R_s - R_s^2}{L_s^2}} = \sqrt{\dfrac{(1000)(120) - (120)^2}{(8.18 \times 10^{-6})^2}} = 3.973 \times 10^7$ rad/sec.

Therefore the circuit will oscillate at any $f > \dfrac{3.973 \times 10^7}{2\pi} = 6.323$ MHz

(b) The most likely frequency of oscillation is at the resonance frequency of L and C

$$\omega_o = \frac{1}{\sqrt{L_s C}} = \frac{1}{\sqrt{(8.18 \times 10^{-6})(56.84 \times 10^{-12})}} = 4.638 \times 10^7 \frac{\text{rad}}{\text{sec}}$$

$f_o = 7.381$ MHz

Note that the circuit $Q = \dfrac{\omega_o L_s}{R_s} = 3.162$ is relatively low and oscillation may occur at other frequencies.

5-5.1

From (5-18) $\alpha_{min} = \dfrac{1}{1 + \frac{C_2}{C_1}} + \dfrac{R_i}{R_t}\left(1 + \dfrac{C_2}{C_1}\right)$

Solving this for R_t, $R_t = \dfrac{R_i (1 + C_2/C_1)}{\alpha_{min} - \dfrac{1}{1 + C_2/C_1}}$

For low R_L, $R_t \approx R_L$. Assume $R_i = 50\,\Omega$

5-5.1 continued

For the circuit of Fig. 5-12, $1 + \frac{C_2}{C_1} = 1 + \frac{2842}{218} = 14.0367$.

Hence $R_L \approx R_t = \frac{701.835}{\alpha_{min} - 0.07124}$.

As α_{min} varies inversely with R_L, the above equation gives the minimum R_L that can be used with a specified α_{min}.

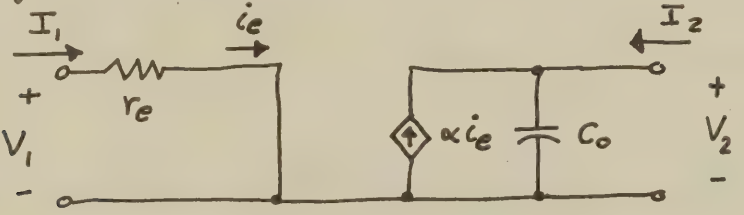
Results:

Part	α_{min}	minimum R_L
a	0.98	772Ω
b	0.99	763Ω
c	0.999	756Ω
d	1.0	756Ω

5-5.2

(a) y parameters of the transistor:

$r_e = 5.3\Omega$
$\alpha = 0.99$
$C_o = 2$ pF



$y_{12}^t = \frac{I_1}{V_2}\Big|_{V_1=0} = 0$

$y_{11}^t = \frac{I_1}{V_1}\Big|_{V_2=0} = \frac{1}{r_e} = 1.887 \times 10^{-1}\,\mho$

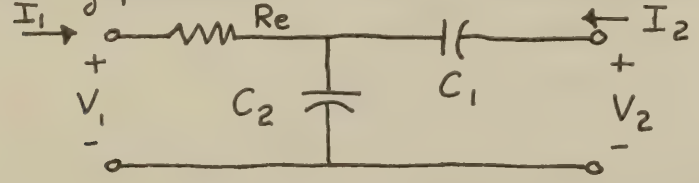
$y_{21}^t = \frac{I_2}{V_1}\Big|_{V_2=0} = \frac{-\alpha i_e}{V_1} = \frac{-\alpha V_1/r_e}{r_e} = \frac{-\alpha}{r_e} = -1.868 \times 10^{-1}\,\mho$

$y_{22}^t = \frac{I_2}{V_2}\Big|_{V_1=0} = j\omega C_o = j\,1.257 \times 10^{-4}\,\mho$

$C = \frac{|y_F y_r|}{2 g_i g_o - \text{Real}\{y_F y_r\}} = \frac{0}{0}$.

This is indeterminant. However since $y_r = 0$ there is no output-to-input feedback path and the transistor is unconditionally stable.

(b) y parameters of the feedback network:



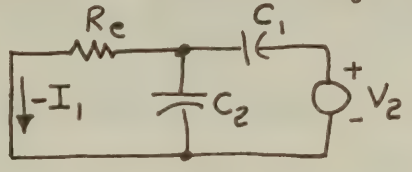
$R_e = 44\Omega$
$C_1 = 218$ pF
$C_2 = 2842$ pF

5-5.2 continued

$$y_{11}^f = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{R_e + \frac{1}{j\omega(C_1+C_2)}} = \frac{1}{44 + \frac{1}{j2\pi\times10^7(2842+218)\times10^{-12}}}$$

$$y_{11}^f = \underline{2.257\times10^{-2}\angle 6.742^\circ} = \underline{2.241\times10^{-2} + j\,2.650\times10^{-3}}\ \mho$$

$$y_{12}^f = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$



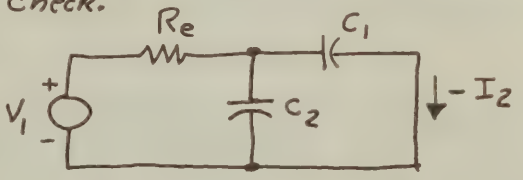
$$y_{12}^f = \left[\frac{-1}{\frac{1}{j\omega C_1} + \frac{R_e(1/j\omega C_2)}{R_e + \frac{1}{j\omega C_2}}}\right] \times \left[\frac{\frac{1}{j\omega C_2}}{R_e + \frac{1}{j\omega C_2}}\right]$$

$$y_{12}^f = \left[\frac{-1}{-j73.01 + \frac{-j246.4}{44 - j5.6}}\right] \times \left[\frac{-j5.6}{44 - j5.6}\right]$$

$$y_{12}^f = (1.274\times10^{-2}\angle 269.5^\circ)\times(0.1263\angle -82.75^\circ) = \underline{1.608\times10^{-3}\angle 186.74^\circ}$$
$$= \underline{-1.597\times10^{-3} - j\,1.887\times10^{-4}}\ \mho$$

For this network, $y_{21}^f = y_{12}^f$, and it is not necessary to calculate y_{21}^f except as a check.

$$y_{21}^f = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$



$$y_{21}^f = \left[\frac{-1}{R_e + \frac{1}{j\omega(C_1+C_2)}}\right] \times \left[\frac{\frac{1}{j\omega C_2}}{\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}}\right] = -y_{11} \times \frac{C_1}{C_1+C_2}$$

$$y_{21}^f = \underline{-1.597\times10^{-3} - j\,1.888\times10^{-4}}\ \mho = \underline{1.608\times10^{-3}\angle 186.74^\circ}$$

5-5.2 continued

$$y_{22}^f = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{\frac{1}{j\omega C_1} + \frac{R_e\,(1/j\omega C_2)}{R_e + 1/j\omega C_2}}$$. This expression was evaluated during the computation of y_{12}^f

$$y_{22}^f = 1.274\times10^{-2}\angle 89.5^\circ = \underline{-1.274\times10^{-2}\angle 269.5^\circ} = \underline{1.112\times10^{-4}} + j\,\underline{1.274\times10^{-2}}\ \mho$$

(c) y parameters of the composite network:

$$[y^n] = [y^t] + [y^f]$$

$$y_{11}^n = 2.111\times10^{-1} + j\,2.650\times10^{-3}\ \mho = \underline{2.111\times10^{-1}\angle 0.7192^\circ}$$

$$y_{12}^n = -1.597\times10^{-3} - j\,1.887\times10^{-4}\ \mho = \underline{1.608\times10^{-3}\angle 186.74^\circ}$$

$$y_{21}^n = -1.884\times10^{-1} - j\,1.887\times10^{-4}\ \mho = \underline{1.884\times10^{-1}\angle -179.94^\circ}$$

$$y_{22}^n = 1.112\times10^{-4} + j\,1.287\times10^{-2}\ \mho = \underline{1.287\times10^{-2}\angle 89.50^\circ}$$

$$C = \frac{|y_f||y_r|}{2g_i g_o - \text{Real}\{y_f y_r\}} = \frac{(1.608\times10^{-3})(1.884\times10^{-1})}{2(2.111\times10^{-1})(1.112\times10^{-4}) - 3.008\times10^{-4}}$$

$$= \frac{3.029\times10^{-4}}{-2.539\times10^{-4}} = -1.193 \Rightarrow \underline{\text{instability}}$$

(d) Calculate K.

$$G_L = \frac{1}{R_t} = \frac{1}{5371\,\Omega\,//\,11{,}300\,\Omega} = 2.747\times10^{-4}\ \mho$$

$$G_s = 0$$

$$K = \frac{2(g_i + G_s)(g_o + G_L)}{|y_f y_r| + \text{Real}\{y_f y_r\}} = \frac{2(2.111\times10^{-1})(1.112\times10^{-4} + 2.747\times10^{-4})}{3.029\times10^{-4} + 3.008\times10^{-4}} = \underline{0.2699}$$

$K < 1$.

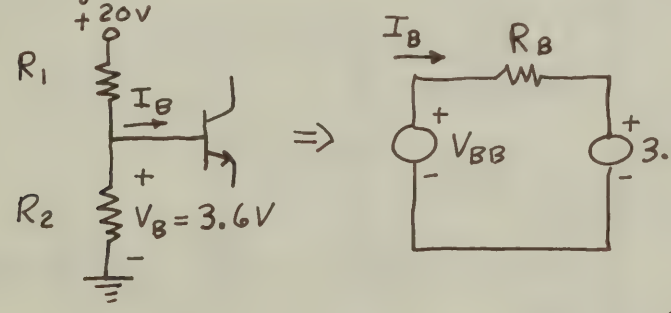
5- 5.3

1. Find I_{CQ} from (5-23). $I_{CQ} = \sqrt{8P_L/R_L} = \sqrt{8\times30\times10^{-3}/4000} = 7.75\,mA$
2. Since $R_o = R_L/2$, $V_{CBQ} = I_{CQ}R_o = I_{CQ}R_L/2 = 15.49\,V.$
3. The transistor must be able to dissipate $P_{dc} = 4P_L = 120\,mW$
   Its minimum f_T should be $2\times3.5\,MHz = 7\,MHz.$
4. Specify R_e. Since $r_e \approx 25/I_{CQ}$ in mA, then $r_e = 2.5/7.75 = 3.23$
   If we choose $R_e = 47\Omega$ (±5%) then $R_i = R_e + r_e$ will be $\approx 50\Omega.$
   R_e must dissipate $I_{CQ}^2 R_e = 3.8\,mW$. Hence a 1/8 or 1/4 W resistor will be more than adequate
5. Find C_s. $C_t = 1/(\omega_o^2 L_t) = 1/[(2\pi\times3.5\times10^6)^2(1.5\times10^{-6})] = 1379\,pF$
   Let $C_f = 135\,pF$ (assume that a 30-300 pF variable capacitor will be used). Then $C_s = C_t - C_o - C_f = 1379 - 4 - 135 = 1240\,pF$
6. Find C_1 and C_2. First calculate $R_p = \omega_o L_t Q_c = 2\pi\times3.5\times10^6\times1.5\times10^{-6}\times160.$
   $R_p = 5278\,\Omega$. Then by (5-30),

$$N = \left[\frac{R_L R_p}{R_i(R_p - R_L)}\right]^{1/2} = \left[\frac{4000 \times 5278}{50(5278-4000)}\right]^{1/2} = 18.18$$

From (5-33) $C_2 = NC_s = 0.0225\,\mu F$. $C_1 = \frac{C_2}{N-1} = 1312\,pF$

7. $R_E + R_e \approx 3/I_{CQ} = 387\,\Omega$. Mathematically $R_E = 337\,\Omega$. The closest standard values are 330Ω and 390Ω; choose 390Ω to be conservative.
   R_E must dissipate $I_{CQ}^2 \times 390 = 23\,mW$. A 1/8 or 1/4 W resistor may be used.
8. Check the power supply voltage requirement. $V_{CC} = V_{CBQ} + I_{CQ}(R_E + R_e)$
   $V_{CC} = 15.49 + 7.75\times10^{-3}(390+50) = 18.9\,V.$ To meet the 20V supply requirement, add a dropping resistor and bypass capacitor (see diagram which follows) whose exact value should be $(20-18.9\,V)/7.75\,mA = 141.94\,\Omega.$
   A 140Ω 1/4 or 1/8 W resistor will do.
9. Specify the base bias resistors.



$$V_{BB} = \frac{20R_2}{R_1 + R_2}$$

$$R_B = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{R_2}{R_1+R_2} = \frac{V_{BB}}{20} = \frac{R_B}{R_1}$$

$$R_1 = \frac{20R_B}{V_{BB}}$$

5-5.3 continued

Choose $R_B = 10(R_e + R_E) \approx 4400\,\Omega$. $V_{BB} = V_B + I_B R_B = 3.6 + \frac{7.75\times10^{-3}\times 4000}{50} = 4.3V$

$R_1 = \frac{4400\times 20}{4.3} = 20.46\,k$. Use $R_1 = 20\,k\Omega$.

$R_2 = \frac{R_1 R_B}{R_1 - R_B} = \frac{20{,}000\times 4400}{20{,}000 - 4400} = 5641\,\Omega$. Use $R_2 = 5600\,\Omega$.

Find the dissipation in R_1 and R_2. R_2 carries $I_{R2} = \frac{3.6}{5600} = 0.643\,mA$

$I_B = 7.75\,mA/50 = 0.155\,mA$.

R_1 carries $I_{R_1} = I_{R_2} + I_B = 0.798\,mA$. The power dissipated by R_1 is $P_{R_1} = 20{,}000\, I_{R_1}^2 = 12.7\,mW$ and the power dissipated by R_2 is $(3.6)^2/5600 = 2.3\,mW$. Hence 1/8 W or 1/4 W resistors can be used for both.

10. As a point of interest we can compute $Q_t = \omega_0 C_t R_0 = 60$. Note that a choice of $L_t > 1.5\,\mu H$ would have lowered C_t and Q_t.

11. Specify the capacitors.

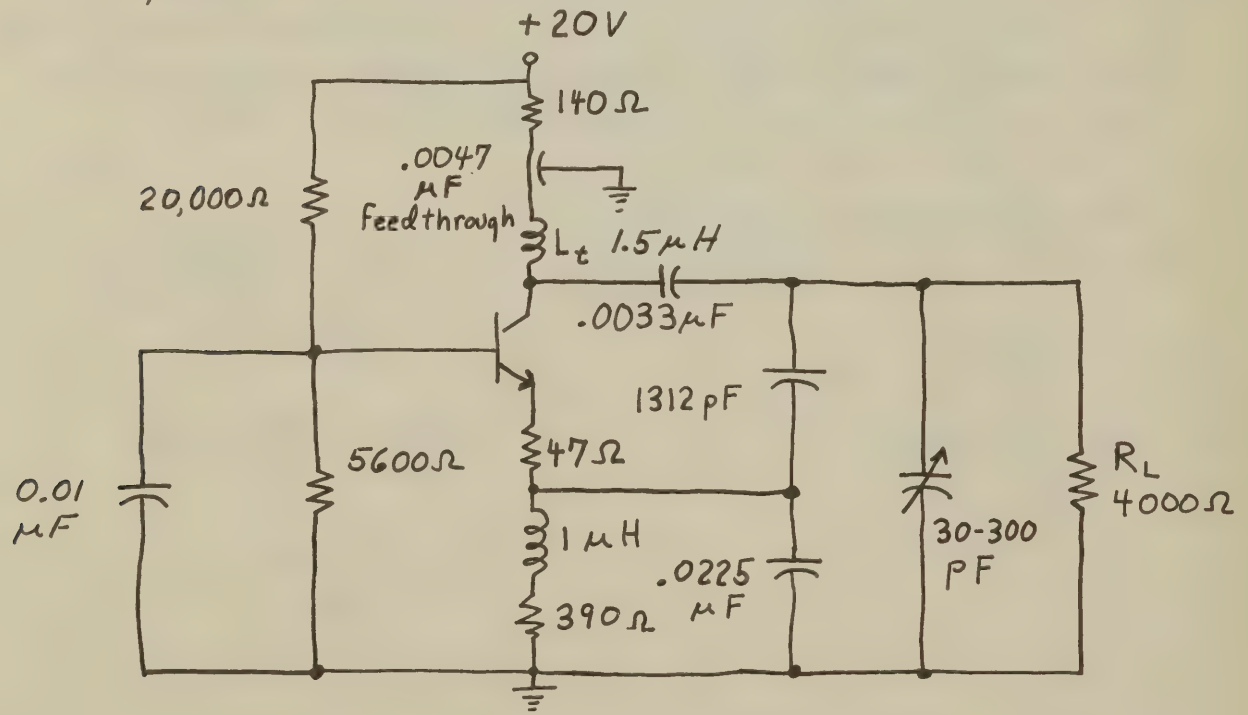
Choose C_B to make $\frac{1}{\omega_0 C_B} << R_i = 50\,\Omega$. IF $\frac{1}{\omega_0 C_B} = 5\,\Omega$,

then $C_B = \frac{1}{5\omega_0} = 9.09\times10^{-9}\,Fd$. Use $C_B = 0.01\,\mu F$.

C_c can be somewhat smaller; $0.0033\,\mu F$ will probably work.

12. Specify the rfc so that $\omega_0 L \geq \frac{10}{\omega_0 C_2} = 20.2$. The minimum value is $0.9186\,\mu H$; use $1\,\mu H$.

The complete circuit is sketched below.



5-5.4

(a) The transistor must be able to dissipate $P_d = 4P_L =$ 100 mW

(b) From (5-23), $I_{CQ} = \sqrt{8P_L/R_L} = \sqrt{(8\times.025)/3000} =$ 8.16 mA.
$R_o = R_L/2 =$ 1500 Ω. $V_{CBQ} = I_{CQ}R_o =$ 12.24 V.

(c) $L_t = 50\times10^{-6}$ Hy. By (5-28), $C_t = 1/[(2\pi F_o)^2 L_t]$.
At $F_o = 1$ MHz, $C_t = 507$ pF, and at $F_o = 2$ MHz, $C_t = 127$ pF.
Hence C_f must provide a total capacitance change of 507-127 or 380 pF. The 270 pF range of the 30 to 300 pF capacitor described in the problem statement is too small.

Note: Some students may attempt to calculate C_t by (5-27), replacing the 50 in the text with the $Q_c = 150$ used in the problem statement. This is wrong because (5-27) involves the loaded Q of the tank circuit; using 150 ignores the loading effects of R_L and N^2R_i.

(d) Let F_1 and F_2 be the upper and lower frequency limits of the oscillator, corresponding to capacitances C_{t_1} and C_{t_2}.

$$C_{t_1} = \frac{1}{(2\pi F_1)^2 L_t} \quad \text{and} \quad C_{t_2} = \frac{1}{(2\pi F_2)^2 L_t}$$

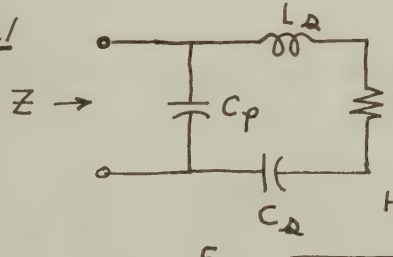
Hence $$\Delta C_t = C_{t_2} - C_{t_1} = \frac{1}{(2\pi F_2)^2 L_t} - \frac{1}{(2\pi F_1)^2 L_t}$$ and

$$L_t = \frac{1}{4\pi^2 \Delta C_t}\left(\frac{1}{F_2^2} - \frac{1}{F_1^2}\right)$$

Decreasing ΔC_t will require increasing L_t. Tuning from 1 to 2 MHz with a ΔC_t of 270 pF requires

$$L_t = \frac{1}{4\pi^2(270\times10^{-12})}\left(\frac{1}{(1\times10^6)^2} - \frac{1}{(2\times10^6)^2}\right) = 70.4\ \mu H$$

5-8.1



At $\omega \to 0$, $Z \to \frac{1}{j\omega C_p}$

At 498.66 kHz, $Z \approx \frac{1}{j\omega C_p}$

Hence $$C_p \approx \frac{1}{j(2\pi\times498.66\times10^3)(-j11{,}000)} =$$ 29 pF

From (5-43), $\frac{F_p}{F_\Delta} = \sqrt{1 + \frac{C_\Delta}{C_p}}$

Hence $$C_\Delta = C_p\left[\left(\frac{F_p}{F_\Delta}\right)^2 - 1\right] = 29\text{pF}\left[\left(\frac{2926.93}{2924.21}\right)^2 - 1\right] =$$ 0.054 pF

At resonance $Z \approx R_s$. Hence $R_s =$ 15 Ω.

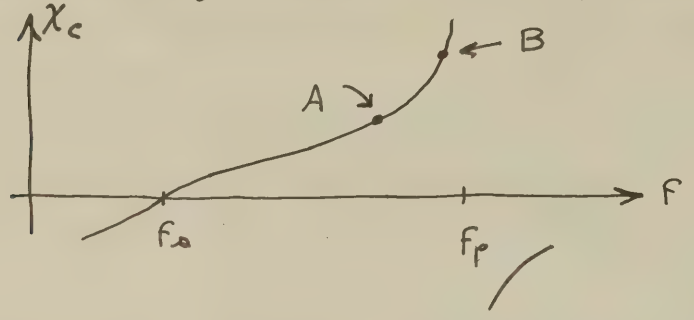
5-8.1 continued

By (5-42), $L_s = \frac{1}{(2\pi f_s)^2 C_s} = \frac{1}{(2\pi \times 2924.21 \times 10^3)^2 \times 0.054 \times 10^{-12}} = \underline{0.055\ Hy}$

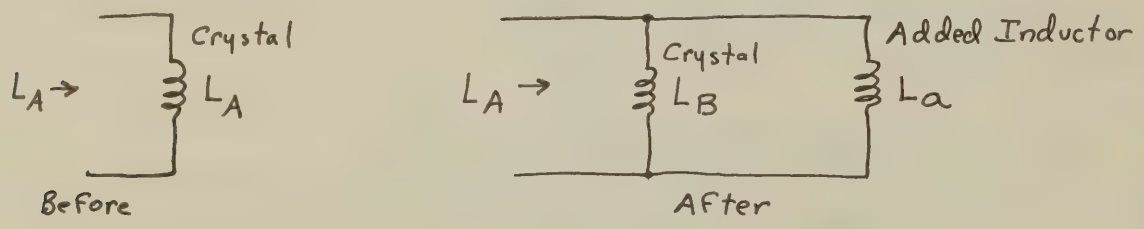
By (5-45), $Q = \frac{2\pi f_s L_s}{R_s} = \frac{2\pi \times 2924.21 \times 10^3 \times 0.055}{15} = \underline{67.4 \times 10^3}$

5-8.2

The frequency dependence of the crystal reactance X_c looks like this:



Initially the crystal was operating at point A and provided an inductance L_A to the oscillator. When the inductor was added in parallel with the crystal, the reactance presented to the rest of the oscillator circuit remained constant. If the frequency change was not large, this meant that the equivalent parallel inductance of the inductor and the crystal had to be the same as the inductance of the crystal at point A.



If $L_B \| L_a = L_A$, then $L_B > L_A$. Hence the crystal reactance had to increase; the crystal moved to an operating point like B above and the frequency increased.

The argument can also be made that the inductor and crystal must produce the same reactance as the crystal alone with less net inductance. This also requires an increase in frequency.

CHAPTER 6
P6-3.1 — 6-3.4

6-3.1

From Example 6-3.1: $K_d = 2$ V/rad, $f_f = 100$ kHz, $K_o = 200\pi$ rad/s/V

(a) $K_a = 50$. $\Delta\omega_H = \pm K_d K_a K_o = \pm 2 \times 50 \times 200\pi = \pm 2\pi \times 10^4$ rad/s.

$\Delta f_H = \pm 10$ kHz.

(b) $k_o = 0.1$ kHz/V; $f_s = 100.5$ kH $= f_f + k_o V_d = 100 + 0.1 V_d$ kHz

$V_d = \frac{0.5}{0.1} = 5$ V ; $V_e = \frac{V_d}{K_a} = 0.1$V ; $\theta_e = \frac{V_e}{K_d} = 0.05$ rad.

6-3.2

The sawtooth detector has $A = 2$ V and $\Delta\omega_H = \pm K_V \theta_{e,max}$.

From Fig. 6-3c, $\theta_{e,max} = \pi$ rad and $K_d = \frac{A}{\theta_{e,max}} = \frac{2}{\pi}$.

$K_V = K_d K_a K_o = \frac{2}{\pi} \times 10 \times 200\pi = 4000$.

$\Delta\omega_H = \pm 4000\pi$ rad/s ; $\Delta f_H = \pm 2$ kHz.

6-3.3

From Example 6-3.2, $K_o = 200\pi$ rad/s/V, $k_o = 100$ Hz/V.

Triangular phase detector has $V_{e,max} = 2$V at $\theta_e = \pi/2$ rad,

or $K_d = 4/\pi$ V/rad $= V_e/\theta_e$.

Amplifier has $K_a = 200$ with max. output $V_{d,max} = \pm 15$V.

∴ The maximum $V_e = \frac{\pm 15}{K_a} = \pm 0.075$ V. This results in

maximum $\theta_e = \frac{\pm 0.075}{K_d} = \pm 0.0589$ rad. $= \pm 3.37°$

Max $\Delta\omega = \pm V_{d,max} \times K_o = 3000\pi$ rad/s.

Max $\Delta f = \pm 1500$ Hz.

6-3.4

From Ex. 6-3.2, $K_d = 4/\pi$, $K_o = 200\pi$. New $K_a = 100$.

Also, $\omega_n = 5000$ rad/s, $\zeta = 0.7$

(a) $\Delta\omega_H = \pm K_V \theta_{e,max} = \pm \frac{4}{\pi} \times 100 \times 200\pi \times \frac{\pi}{2} = \pm 40{,}000\pi$ rad/s.

$\Delta f_H = \pm 20$ kHz.

(b) From (6-25), $\Delta\omega_p = \pm 32{,}880$ rad/s

$\Delta f_p = \pm 5233$ Hz

Note: From (6-24) the lock-in range is

$\Delta\omega_L = \pm 14000$ rad/s or $\Delta f_L = \pm 2228$ Hz.

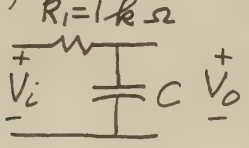
6-3.5

(a) Triangular phase detector has $V_{e,max} = 0.3\,V$ at $\theta_{e,max} = \frac{\pi}{2}$ rad.
The VCO has $f_f = 100\,MHz$, $f_o = (100 + 15V_d)\,MHz$.
$k_o = 15\,MHz/V$.
$K_o = 2\pi k_o = 30\pi \times 10^6$ rad/s/V.

$$K_d = \frac{V_e}{\theta_e} = \frac{0.3}{\pi/2} = \frac{0.6}{\pi} = 0.191\ V/rad.$$

$\Delta f_H = \pm 5\times10^6$; $\Delta\omega_H = \pm 10^7\pi$ rad/s.
From (6-21), $K_V = \frac{\Delta\omega_H}{\theta_{e,max}} = 2\times10^7$; $K_a = \frac{K_V}{K_d K_o} = 1.11$

(b)



$$\frac{V_o}{V_i} = \frac{1}{1+\tau_1 s} = \frac{1}{1+j\omega R_1 C}$$

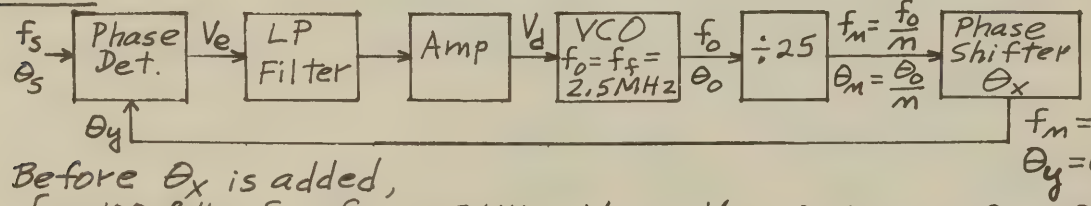
The response is down 3db at $\omega = \frac{1}{R_1 C}$

$$C = \frac{1}{\omega R_1} = \frac{1}{2\pi\times10^5\times10^3} = 1.59\,mF$$

(c) $f_s = 99\,MHz = f_o$; $\therefore 15V_d = -1$, $V_d = -\frac{1}{15} = -0.067\,V$

$$V_e = \frac{V_d}{K_a} = -0.06\,V; \quad \theta_e = \frac{V_e}{K_d} = -0.1\pi \text{ rad.}$$

6-7.1



Before θ_x is added,
$f_s = 100\,kHz$, $f_o = f_f = 2.5\,MHz$, $V_d = 0$, $V_e = 0$, $\theta_e = 0$, $\theta_y = \theta_m = \frac{\theta_o}{n}$
where $n = 25$ is the divider constant. When θ_x is introduced, there is a momentary change in f_o causing θ_o to change. In the steady state, if f_s is constant, f_o, V_d, V_e, θ_e, and θ_y must have the same values as before. Therefore the change in θ_m must be the negative of θ_x, ($\Delta\theta_m = -\theta_x$), and $\Delta\theta_o = -n\theta_x$. f_o is unchanged.
If the loop can track changes in θ_x fast enough, this system can be used as a phase modulator.

6-7.2

$\Delta\theta_o = 10 \times \Delta\theta_m = 1$ radian

CHAPTER 7
P7-1.1 — 7-1.3

7-1.1

$$i_o(t) = 11 + 5.75\cos\omega t + \cos 2\omega t + 0.25\cos 3\omega t \ \text{mA}.$$

7-1.2

f	i_o (mA)
0	11.25
50	0.375
350	1.00
400	6.125
750	3.344
800	1.00
1100	0.187
1150	1.00
1200	0.250
1500	0.250
1550	0.375
1900	0.187
2250	0.031

Note: The solution can be obtained by evaluating terms from Table 7-1 or by means of graphical convolution. In the graphical method, the spectrum of $V_i^2(f) = V_i(\lambda) * V_i(f-\lambda)$ (convolution) is found first. Then the spectra of $V_i^2(\lambda)$ and $V_i(f-\lambda)$ are convolved to obtain $V_i^3(\lambda)$.

7-1.3

Start with the Fourier transforms,

$$G(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \quad (1), \text{ and } f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(j\omega)e^{j\omega t}d\omega \quad (2).$$

Change the variable of integration to $f = \omega/2\pi$. Then $d\omega = 2\pi\, df$ and (2) can be written as

$$f(t) = G(f)e^{j2\pi tf}df \quad (3).$$ [This is equation (7-6).]

(a) If $G(f) = \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$ where "δ" is the delta function, substitution into (2) yields

$$f(t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} = \cos\omega_0 t.$$

Similarly, if $G(f) = 0.5[\delta(f-f_0) + \delta(f+f_0)]$ is substituted into (3), the result is $f(t) = \cos\omega_0 t$.

(b) $f(t) = A\cos(\omega_0 t + \theta) = \frac{A}{2}[e^{j\theta}e^{j\omega_0 t} + e^{-j\theta}e^{-j\omega_0 t}]$

By analogy from the result in part (a),

$$G(f) = 0.5A[e^{j\theta}\delta(f-f_0) + e^{-j\theta}\delta(f+f_0)].$$

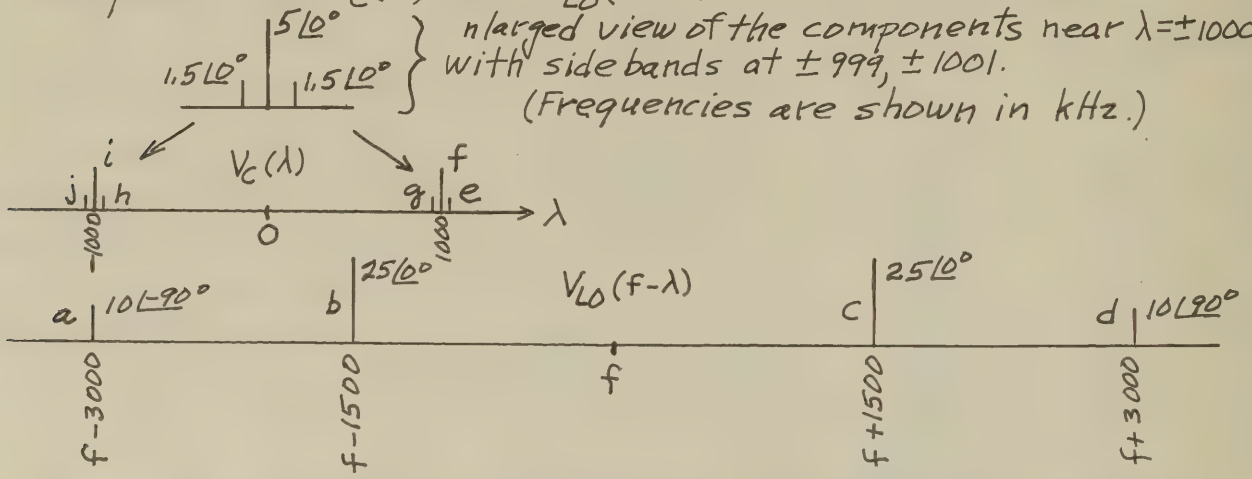
(c) $f(t) = \sin\omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = \frac{e^{-j\pi/2}}{2}[e^{j\omega_0 t} - e^{-j\omega_0 t}]$

$$G(f) = \frac{e^{-j\pi/2}}{2}[\delta(f-f_0) - \delta(f+f_0)] = 0.5[e^{-j\pi/2}\cdot\delta(f-f_0) + e^{j\pi/2}\delta(f+f_0)]$$

7-1.4

$v_o(t) = v_c(t) \times v_{LO}(t) = [10\cos 2\pi\times1000\times10^3 t + 3\cos 2\pi\times1001\times10^3 t + 3\cos 2\pi\times999\times10^3 t] \times [50\cos 2\pi\times1500\times10^3 t + 20\sin 2\pi\times3000\times10^3 t]$

The spectra for $V_c(\lambda)$ and $V_{LO}(f-\lambda)$ are shown here.



The table gives values of the products of various components as f is varied. Only the values for the positive-frequency terms are found. Amplitudes of the time-domain terms are just double these values (because of symmetry) and the angles are the same as those of the positive-frequency terms.

Product Terms	f (kHz)	Product value	Angle	Time-domain amplitude
a x e	4001	15 μV	-90°	30 μV
a x f	4000	50	-90°	100
a x g	3999	15	-90°	30
b x e	2501	37.5	0°	75
b x f	2500	125	0°	250
b x g	2499	37.5	0°	75
a x h	2001	15	-90°	30
a x i	2000	50	-90°	100
a x j	1999	15	-90°	30
b x h	501	37.5	0°	75
b x i	500	125	0°	250
b x j	499	37.5	0°	75

7-1.5

Let $v_i(t) = f_1(t) + f_2(t)$ and $i_o(t) = b(v_i(t))^2$

$f_1(t) = 10\cos\omega_c t + 3[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t]$ (mV)

$f_2(t) = 100\cos\omega_{LO}t + 20\cos 2\omega_{LO}t$ (mV)

$f_c = 1500$ kHz, $f_c + f_m = 1502$ kHz, $f_c - f_m = 1498$ kHz,

$f_{LO} = 1955$ kHz, $2f_{LO} = 3910$ kHz.

In the output current, the product term is

$i_o(t) = 2b \times f_1(t) \times f_2(t)$, with $b = 5\times10^{-3}$, $2b = 10^{-2}$.

By use of graphical convolution, the values of the components in $i_o(t)$ are found.

f (kHz)	i_o amplitude (μA)	
5412	0.3	
5410	1.0	
5408	0.3	
3457	1.5	
3455	5.0	
3453	1.5	
2412	0.3	
2410	1.0	
2408	0.3	
457	1.5	Desired difference-frequency terms.
455	5.0	
453	1.5	

7-3.1

The wave has even symmetry about $t=0$, hence the Fourier series contains only cosine terms.

$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t$, where $\omega = 2\pi/T$

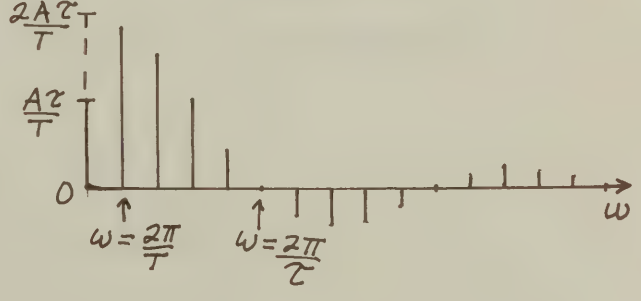
By inspection, $a_0 = A\tau/T$.

$a_n = \frac{2}{T}\int_0^{\tau/2} f(t)\cos n\omega t\,dt = \frac{4}{T}\int_0^{\tau/2} A\cos n\omega t\,dt$

$a_n = 2A\frac{\tau}{T}\,\frac{\sin n\pi\tau/T}{n\pi\tau/T}$

(a) Spacing between spectral lines is $f = \frac{1}{T}$ or $\omega = \frac{2\pi}{T}$

(b) Zero crossings occur at $f = \frac{1}{\tau}$ or $\omega = \frac{2\pi}{\tau}$



P7-5.1 — 7-6.2

7-5.1

Use the relations $v_i^3 = (v_1+v_2)^3 = v_1^3 + 3v_1^2v_2 + 3v_1v_2^2 + v_3^3$

$$v_1^2 = \frac{V_1^2}{2}(1+\cos 2\omega_1 t)$$

$$v_2^2 = \frac{V_2^2}{2}(1+\cos 2\omega_2 t)$$

$$v_1^3 = \frac{V_1^3}{4}(\cos 3\omega_1 t + 3\cos\omega_1 t)$$

and so forth, to arrive at the terms in the table.

7-6.1

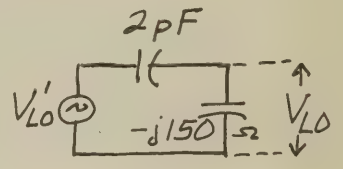
$g_{mo} = 6$ mU, $V_p = -3$V. Bias at $V_{GS} = -1.5$V.
The peak value of L.O. voltage between gate and source is $V_{LO} = 1.5$ V.

(a) From (7-30), $g_c = \frac{g_{mQ} V_{LO}}{|V_p|} = \frac{g_{mo}}{2} \times \frac{V_{LO}}{|V_p|} = \underline{1.5\ \text{mU}}$

(b) From (7-29), $I_{IF} = g_c \times V_{RF} = \underline{1.5\ \text{mA}}$.

(c) At $f_{LO} = 115$ MHz, the 2pF capacitor has 692 Ω reactance. Let V'_{LO} be the voltage at the L.O. terminal

$$V'_{LO}(\text{rms}) = \frac{1.5}{\sqrt{2}} \times \frac{150+692}{150} = \underline{5.95\ \text{V}}$$



7-6.2

For the $v_{GS}(t)$ in Fig. 7-12b, $g_m(t)$ in Fig. 7-12c is given by (7-34) for $v_{GS} > V_p$, and $g_m(t) = 0$ for $v_{GS} < V_p$.
$v_{GS}(t) = 0$ at $\omega_{LO}t = \theta_c$ given by (7-31).
Fourier Analysis:

$$g_{m1} = \frac{2}{\pi}\int_0^{\theta_c} g_m(t)\cos\theta\, d\theta, \text{ where } \theta = \omega_{LO}t.$$

$$g_{m1} = \frac{2g_{mo}}{\pi|V_p|}\int_0^{\theta_c}[V_{LO}\cos^2\theta + (V_{GS}-V_p)\cos\theta]\,d\theta$$

Integration yields (7-35).

7-6.3

$V_P = -4\,V$, $g_{m0} = 10\,m\mho$, $V_{RF} = 10\,\mu V$

(a) $V_{GS} = -3\,V$, $V_{LO} = 3\,V$

From (7-31), $\theta_c = \cos^{-1}\left[\frac{V_P - V_{GS}}{V_{LO}}\right] = 109.5^\circ$ or 1.91 rad.

From (7-35), $g_{m1} = 5.31\,m\mho$

From (7-38), $g_c = \frac{g_{m1}}{2} = 2.65\,m\mho$

and $I_{IF} = \frac{g_{m1}}{2} \times V_{RF} = 26.5\,nA$

(b) $V_{GS} = -4\,V$, $V_{LO} = 4\,V$.

$\theta_c = 90^\circ$ or $\pi/2$ rad

$g_{m1} = 5\,m\mho$

$g_c = 2.5\,m\mho$

$I_{IF} = 25\,nA$

(c) $V_{GS} = -5\,V$, $V_{LO} = 5\,V$

$\theta_c = 78.4^\circ$ or 1.37 rad.

$g_{m1} = 4.67\,m\mho$

$g_c = 2.33\,m\mho$

$I_{IF} = 23.3\,nA$

CHAPTER 8

P8-1.1 — 8-1.2

8-1.1

(a) $m_a = \frac{14}{28} = \underline{0.5}$

(b) Carrier power $P_c = 10\,W = \frac{V_c^2}{2\times75}$; $V_c = 38.73\,V$

Using (8-8), the power in the two side frequencies is $P_{sb} = \left(\frac{V_c m_a}{2}\right)^2 \times \frac{1}{2\times75} \times 2 = 1.25\,W$

$P_T = P_c + P_{sb} = \underline{11.25\,W}$

(c) From part (b), $P_T = P_c + P_{sb} = \frac{V_c^2}{2\times75}\left(1 + \frac{m_a^2}{4}\times 2\right) = P_c\left(1+\frac{m_a^2}{2}\right)$

(d) (1) Carrier voltage (unmodulated) $= V_c = \underline{38.73\,V}$

(2) Peak voltage in modulated wave is $V_c(1+m_a) = \underline{58.1\,V}$

(e) With $V_m = 28\,V$, $m_a = 1$. RF voltage at the modulation peak is $2V_c$. $\underline{PEP} = \frac{(2V_c)^2}{2\times75} = \underline{40\,W}$

(f) $v_m(t) = 8.4\cos(2\pi\times500t) + 2.8\sin(2\pi\times755t)$

Let $m_{a1} = \frac{8.4}{28} = 0.3$, $m_{a2} = \frac{2.8}{28} = 0.1$

$f_{m1} = 500\,Hz$ $f_{m2} = 755\,Hz$

Use (8-6) and (8-8) to get

$v(t) = 38.73\cos\omega_c t + 5.81\cos(\omega_c+\omega_{m1})t + 5.81\cos(\omega_c-\omega_{m1})t + 1.94\sin(\omega_c+\omega_{m2})t - 1.94\sin(\omega_c-\omega_{m2})t.$

$\underline{P_T} = \left[\frac{(38.73)^2}{2} + \frac{(5.81)^2}{2}\times2 + \frac{(1.94)^2}{2}\times2\right]\frac{1}{75} = \underline{10.5\,W}$

The spectrum has lines at 399.245, 399.5, 400, 400.5, and 400.755 kHz.

(g) At some time t, the two modulating waves will peak together, giving a maximum modulating voltage of $8.4+2.8 = 11.2\,V$. Then $\underline{m_{a,max}} = \frac{11.2}{28} = \underline{0.4}$

8-1.2

(a) $i(t) = 10\cos\omega_c t + 4\cos(\omega_c+\omega_m)t + 4\cos(\omega_c-\omega_m)t$ mA

10 mA at $f_c = 1000$ kHz

4 mA " $f_c + f_m = 1005$ kHz

4 mA " $f_c - f_m = 995$ kHz

(continued on next page)

8-1.2 (continued)

(b) At ω_c, $R_t = 1000\,\Omega$, $Q_t = 100 = \frac{R_t}{\omega_c L} = R_t \omega_c C$

$\omega_c L = 10\,\Omega$, $\omega_c C = 0.1\,\mho$

At $f_c + f_m = 1005$ kHz, $Y_t = \frac{1}{R_t} + j\left(\omega C - \frac{1}{\omega L}\right) = 0.001 \times \sqrt{2} \angle 45°$

$Z_t = \frac{1}{Y_t} = 707.1 \angle -45°$

At $f_c - f_m = 995$ kHz, $Z_t = 707.1 \angle 45°$

Voltage (peak) at f_c is 10 V.

" " " $f_c \pm f_m$ is 2.828 V

(c) m_a for the current wave is 0.8 or 80%

m_a " " voltage " " 0.566 or 56.6%

8-1.3

If Q_t is decreased, m_a will increase because $|Z_t|$ will be larger at the side frequencies.

8-1.4

Assume that the circuit is tuned by varying C. It is now resonant at $f_o = 1005$ kHz.

Let $f_u = f_o = 1005$ kHz, $f_c = 1000$ kHz, $f_L = 995$ kHz.

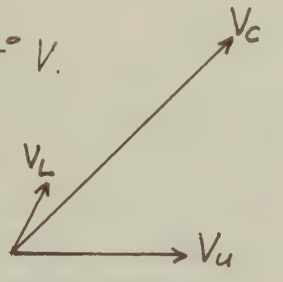
From P8-1.2, $\omega_c L = 10\,\Omega$, hence at f_u, $\omega_u L = 10.05\,\Omega$ and $\omega_u C = 0.0995\,\mho$.

At $f_u = 1005$ kHz, $Z_t = 1000 \angle 0°$, $V_u = 4 \times 10^{-3} \times 1000 = 4 \angle 0°$ V.

" $f_c = 1000$ kHz, $Z_t = 707 \angle 45°$, $V_c = 7.07 \angle 45°$ V.

" $f_L = 995$ kHz, $Z_t = 447 \angle 63.4°$, $V_L = 1.79 \angle 63.4°$ V.

The side-frequency components have unequal amplitudes and make unequal angles with respect to the carrier. The resultant will exhibit both amplitude and phase modulation.



8-3.1

Modulator A: Input – $v_A = V_c \cos \omega_c t + V_m \cos(\omega_m t + \theta)$

Output – $i_{oA} = a v_i^2 = \frac{a V_c^2}{2}(1 + \cos 2\omega_c t) + \frac{a V_m^2}{2}(1 + \cos(2\omega_m t + 2\theta))$

$+ a V_c V_m \left[\cos(\omega_c t + \omega_m t + \theta) + \cos(\omega_c t - \omega_m t - \theta)\right]$

(continued on next page)

8-3.1 (continued)

Modulator B: Input – $v_B = V_c \cos(\omega_c t + 90°) + V_m \cos(\omega_m t + 90°)$

Output – $i_{OB} = \frac{a V_c^2}{2}\left[1 + \cos(2\omega_c t + 180°)\right] + \frac{a V_m^2}{2}\left[1 + \cos(2\omega_m t + 180°)\right]$
$+ a V_c V_m \left[\cos(\omega_c t + \omega_m t + 180°) + \cos(\omega_c t - \omega_m t)\right]$

When the sideband components of i_{OA} and i_{OB} are added,

$i_O = 2a V_c V_m \cos(\omega_c - \omega_m)t$ [with $\theta = 0$]

Only the lower sideband remains. The 2nd harmonic and upper sideband components cancel.

8-3.2

Let Mod. A input be $v_A = V_c \cos \omega_c t + V_m \cos \omega_m t$

and Mod B input be $v_B = V_c \cos(\omega_c t + 90° + \phi) + V_m \cos(\omega_m t + 90°)$

The sideband output components become:

$i_{OA} = a V_c V_m \left[\cos(\omega_c t + \omega_m t) + \cos(\omega_c t - \omega_m t)\right.$

$i_{OB} = a V_c V_m \left[\cos(\omega_c t + \omega_m t + 180° + \phi) + \cos(\omega_c t - \omega_m t + \phi)\right]$

Upper sideband components do not completely cancel and lower sideband output is reduced if $\phi \neq 0$.

8-3.3

Let Mod. B input be $v_B = V_c \cos(\omega_c t + 90°) + V_m \cos(\omega_m t + 90° + \phi)$

Then the sideband output from Mod. B is

$i_{OB} = a V_c V_m \left[\cos(\omega_c t + \omega_m t + 180° + \phi) + \cos(\omega_c t - \omega_m t - \phi)\right]$

The result is the same as in P8-3.2.

8-3.4

The mixer input is $v_i = V_c \cos(\omega_c t + \phi) + 2a V_c V_m \cos(\omega_c t - \omega_m t)$

Assume $i_O = v_i^2$ and consider only the difference – frequency term.

$i_O = 2a V_c^2 V_m \cos\left[\omega_c t + \phi - (\omega_c t - \omega_m t)\right] = 2a V_c^2 V_m \cos(\omega_m t + \phi)$

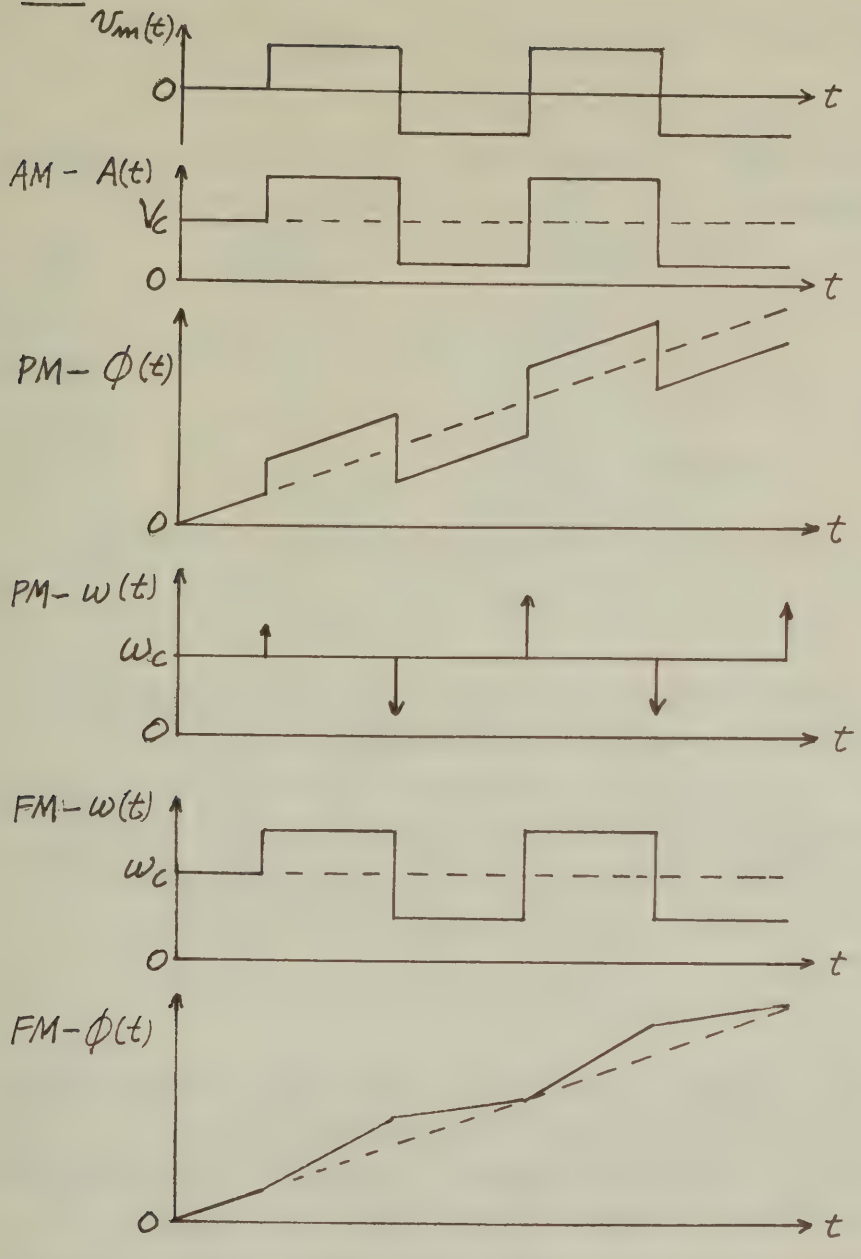
Angle ϕ is added to all audio frequencies. The waveshape would be altered, but the human ear would probably not detect the difference for small ϕ.

8-3.5

Mixer output would be $i_O = 2a V_c^2 V_m \cos(\omega_m + \Delta\omega)t$.

All audio frequencies shifted by $\Delta\omega$. This would destroy the harmonic relationships in speech and (especially) music.

8-4.1



8-4.2

(a) $\Delta f_0 = 10 \times 12 = 120$ kHz.

(b) $f_0 = 60$ MHz, $\Delta f_0 = 10$ kHz

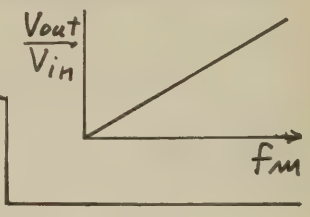
(c) $m_f = \dfrac{\Delta f_0}{f_m} = 10$

(d) $\Delta f_0 = 20$ kHz, $m_f = \dfrac{20}{0.5} = 40$

8-4.3

In FM, $m_f = \frac{\Delta f}{f_m}$ with Δf proportional to V_m.

In PM, $\Delta\theta = m_p$ is proportional to v_m.
Hence, to produce PM the "effective V_m" into the FM modulator must increase with f_m. The audio signal must be differentiated to make $V_{out} \propto \omega \times V_{in}$.



8-4.4

(a) $\Delta\theta = m_f = \frac{\Delta f}{f_m} = 100$ rad.

(b) $B \approx 2(\Delta f + f_m) = 101$ kHz

(c) $\Delta f = 10$ kHz

(d) $\Delta f = 50$ kHz, $\Delta\theta = 20$ rad, $B \approx 2(50+2.5) = 105$ kHz

---

8-4.5

Modulate the FM signal generator with an audio signal of amplitude V_m and frequency f_m. Measure f_m with an accurate counter. As V_m is increased from zero, use the spectrum analyzer to determine points where the carrier or selected side-frequency pairs disappear. Use Fig. 8-15 or a Bessel table to determine m_θ. Then $\Delta f = m_\theta \times f_m$.

8-4.6

Modulate the transmitter with a constant V_m, variable f_m. Observe the output on the spectrum analyzer.
As f_m is increased, the <u>number</u> of significant spectral lines will remain the same for PM, decrease for FM.

8-5.1

Refer to equations (8-34) and (8-36), with $V_c = 1$ V.
Assume the voltage is applied to $R = 1\ \Omega$.
The total power in the wave is $P = V_c^2/2R = 0.5$ W

(a) For $m_\theta = 5$, 6 side-frequency pairs contain 99.4% of energy.
10 " " " " ≈100% " "

(b) For $m_\theta = 10$, 6 " " " " 40.7% " "
10 " " " " 96.0% " "

Bessel amplitudes are tabulated on the next page.

8-5.1 (continued)

Amplitudes of the Bessel Functions.

	$m_\theta = 5$	$m_\theta = 10$
J_0	-0.1776	-0.2459
J_1	-0.3276	0.0435
J_2	0.0466	0.2546
J_3	0.3648	0.0584
J_4	0.3912	-0.2196
J_5	0.2611	-0.2341
J_6	0.1310	-0.0145
J_7	0.0534	0.2167
J_8	0.0184	0.3179
J_9	0.0055	0.2919
J_{10}	0.0015	0.2075

Use these values in (8-36) to obtain the power.

8-5.2

(a) $P = \frac{V_c^2}{2R} = \frac{100}{2\times 10} = 5\,W$

(b) $P = 5\,W$ for any value of m_θ.

(c) For $m_\theta = 5$, number of side-freq. pairs = 5
" $m_\theta = 20$, " " " " " " = 19

(d) For $m_\theta = 5$, $B = 10\,kHz$
" $m_\theta = 20$, $B = 38\,kHz$

8-8.1

Note: Determination of bandwidths for pulse-modulated waves is not treated in this text. The solutions offered for P8-8.1, 8.2, and 8.3 yield approximate answers. More sophisticated approaches can be found in other books.

(a) $B = 4\times 24 = \underline{96\,kHz}$.

(b) (1) Sampling rate $f_s = 7\,kHz$ minimum. $\underline{f_s = 8\,kHz}$ is typically used.

(2) Pulses or bits/s $= 24\times 8\times 10^3 = \underline{192\times 10^3}$.

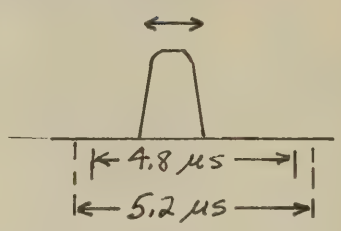
(3) The time allotted to each pulse is $\tau = \frac{1}{192{,}000} = 5.2\,\mu s$.

If rise and fall times for the pulses are assumed to be $1\,\mu s$, the bandwidth is approximately $B = \frac{1}{10^{-6}} = 10^6\,Hz = \underline{1\,MHz}$.

8-8.2

(a) Time interval for each pulse is 5.2 μs as in P8-8.1.

(b) Let the pulse shift within a 4.8 μs region, leaving a 0.2 μs guard band on either side.
If the pulse width is 1.4 μs, the allowable time shift from center position is $\frac{4.8-1.4}{2} = \pm 1.7\ \mu s$



(c) If the pulse has rise and fall times of 0.4 μs,
$B \approx \frac{1}{t_r} = 2.5\ MHz$.
This is greater than the bandwidth for PAM because the pulse width and rise time are smaller.

8-8.3

(a) Bit rate is equal to
(number of channels) × (sampling rate) × (bits/sample)
$= 96 \times 8 \times 10^3 \times 8 = 6.144 \times 10^6$ bits/s.

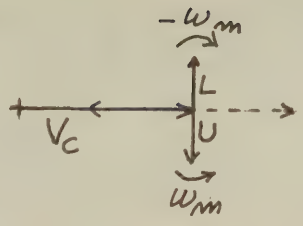
(b) A possible "worst condition" occurs if there are alternate pulses and spaces. The fundamental-frequency component of this square wave is $\frac{6.144 \times 10^6}{2} = 3.07\ MHz$
This represents the ideal minimum bandwidth required.

CHAPTER 9

P9-7.1 — 9-7.2

9-7.1

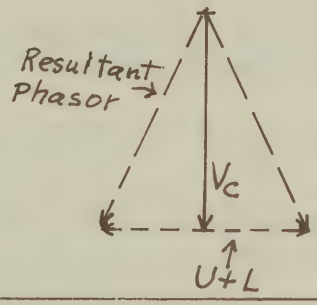
(a) The carrier (V_c) and side-frequency phasors are shown at $t=0$. U (upper side freq.) and L (lower side freq.) produce a resultant in phase with V_c.



If $V_c > 2V_s$ an envelope detector will recover the modulating wave.

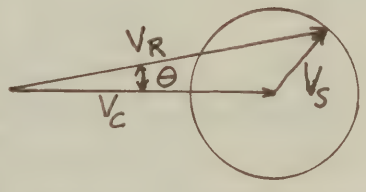
A different phase of V_c relative to the side-freqs. will result in distortion.

(b) The sum of U and L adds at right angles to the reinserted carrier. The resultant phasor exhibits both phase and amplitude modulation. The amplitude variation is at frequency $2f_m$.



9-7.2

(a) The phasor V_s rotates about the tip of V_c at the difference frequency ω_m. V_R is the resultant. Envelope detector output is $\approx |V_R|$.



For $V_c >> V_s$, the output wave is nearly sinusoidal.

For $V_c \approx V_s$, the output approaches the form of a rectified sine wave. See (b).

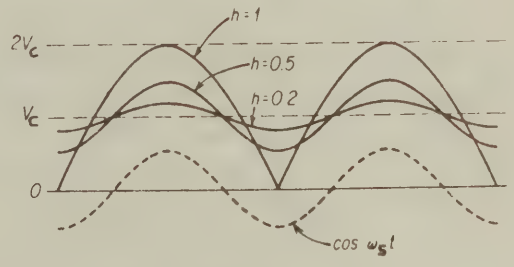
(b) In the phasor diagram above let $V_R = A + jB$.

$A = V_c + V_s \cos \omega_m t$; $B = V_s \sin \omega_m t$; $|V_R| = [A^2 + B^2]^{1/2}$,

$\theta = \tan^{-1} B/A$. Substitution of $h = V_s/V_c$ yields

$|V_R| = V_c [1 + 2h \cos \omega_m t + h^2]^{1/2}$ and $\theta = \tan^{-1} \left[\frac{h \sin \omega_m t}{1 + h \cos \omega_m t} \right]$

Since the entire phasor diagram rotates with $\omega_c t$, the desired result follows.



9-7.3

(a) $C_T = 180\ pF$, $k = 0.95$, $f_o = 455\ kHz$, $B = 20\ kHz$.
$Q_t = f_o/B = 22.75$.
From Table 3-8: $L_1 = \frac{1}{\omega_o^2 C_T} = \underline{680\ \mu H}$; $R_t = \frac{1}{2\pi B C_T} = 44.2\ k\Omega$
$Q_p = k^2 Q_t = 20.53$; $L_2 = \frac{5000}{\omega_o Q_p} = \underline{85.2\ \mu H}$
$M = \frac{L_1}{N} = L_1\sqrt{\frac{5\ k\Omega}{44.2\ k\Omega}} = \underline{229\ \mu H}$
Since an input impedance of $20\ k\Omega$ is desired on the primary side, use (3-30) to determine that the tap position should be at $\approx \underline{67\%}$ of the total turns on the L_1 coil.

(b) Let the secondary load resistance be $R_L = 5\ k\Omega$.
From (9-13), $R_L \approx \frac{R_1}{2M}$ or $\underline{R_1 = 10\ k\Omega}$.

(c) From (9-20), $\underline{C_1 \le 15.42\ mF}$

(d) Use (9-25) with $R_m = 10k \| 50k = 8333\ \Omega$, $|Z_m| = 7729\ \Omega$,
$\underline{m_a \le \frac{7729}{10000} = 0.773}$

9-8.1

The input to the product detector is equivalent to
$v_i(t) = V_L \cos(\omega_c - \omega_m)t + V_C \cos[(\omega_c + \delta\omega)t + \phi]$.
The desired difference-frequency output term is
$v_o(t) = V_L V_C \cos[(\omega_c + \delta\omega)t - (\omega_c - \omega_m)t + \phi]$
$= V_L V_C \cos[(\omega_m + \delta\omega)t + \phi]$

(a) Angle ϕ is added to all output frequency components. For voice transmission, the quality would not be affected.

(b) All frequencies in the demodulated signal shifted by $\frac{\delta\omega}{2\pi}$ Hz. Harmonic relationships would be destroyed.

9-8.2

Let $v_C(t) = V_C \cos[(\omega_c + \delta\omega)t + \phi]$,
$v_U(t) = V_U \cos(\omega_c + \omega_m)t$, and $v_L(t) = V_L \cos(\omega_c - \omega_m)t$
be the inputs to a square-law device. If $V_U = V_L = V_S$, the difference-frequency output components will be proportional to:
$v_o(t) = V_C V_S \cos[(\omega_m - \delta\omega)t - \phi] + V_C V_S \cos[(\omega_m + \delta\omega)t + \phi]$
$+ V_S^2 \cos 2\omega_m t$.

(continued on next page)

9-8.2 (continued)

The third term represents 2d-harmonic distortion that is present in all cases. Make $V_c >> V_s$ to minimize this.

(a) $\delta\omega = 0$, $\phi = 0$.
The desired output is $2V_c V_s \cos \omega_m t$.
The LP filter may not eliminate the 2d-harmonics.

(b) $\delta\omega = 0$, $\phi \neq 0$.
The desired output is $V_c V_s [\cos(\omega_m t - \phi) + \cos(\omega_m t + \phi)]$
$= 2V_c V_s \cos \omega_m t \times \cos \phi$.
This becomes zero if $\phi = \pi/2$.

(c) $\phi = 0$, $\delta\omega \neq 0$.
The output becomes $V_c V_s [\cos(\omega_m + \delta\omega)t + \cos(\omega_m - \delta\omega)t]$
-two unequal frequencies.

9-8.3

This represents "product detection" of an AM wave. Since the received carrier and the locally-injected one may be of quite different amplitudes and the resulting mixer-product coefficients may also differ, let $c_I = V_{CI} \cos(\omega_c t + \delta\omega t + \phi)$ be the injected carrier,
$v_{CR} = V_{CR} \cos \omega_c t$ the received carrier.

(a) The desired output will have components proportional to $2V_s V_{CR} \cos \omega_m t$ and $2V_s V_{CI} \cos \omega_m t$.

(b) Components $2V_s V_{CR} \cos \omega_m t$ and $2V_s V_{CI} \cos\phi \cos \omega_m t$.

(c) Components $2V_s V_{CR} \cos \omega_m t$, $2V_s V_{CI} \cos(\omega_m + \delta\omega)t$,
and $2V_s V_{CI} \cos(\omega_m - \delta\omega)t$.

As in the previous case (P9-8.2), it is essential that the locally-injected carrier be exactly correct in frequency to avoid distortion.

CHAPTER 10
P 10-1.1 — 10-1.3

10-1.1

The detector requires 1 millivolt across 40 kΩ or 2.5×10^{-11} W. This means that 1 millivolt also appears across the 2000Ω filter termination. The power lost in the termination is $(1\text{ mV})^2/2000\Omega$ or 5×10^{-10} W. The total power out of the filter is $5 \times 10^{-10} + 2.5 \times 10^{-11} = 5.25 \times 10^{-10}$ W. An insertion loss of 3.5 dB corresponds to a power gain of $10^{-.35} = 0.4467$. Power input to the filter $= 5.25 \times 10^{-10}/0.4467 = 1.175 \times 10^{-9}$ W. Power input to the RF amplifier was already specified as 1.333×10^{-14} W.

$$\text{Gain required} = 10 \log_{10}\left\{\frac{1.175 \times 10^{-9}}{1.333 \times 10^{-14}}\right\} = \underline{49.5\text{ dB}}.$$

10-1.2

Consider a network which introduces a frequency-independent time delay of τ seconds and which has a constant gain of A. If the input voltage to the network is $v_i(t)$ for $t > 0$, then the output voltage $v_o(t) = A\, v_i(t-\tau)\, u(t-\tau)$, where $u(t-\tau)$ is the unit step function. In terms of Laplace transforms,

$$V_o(s) = \mathcal{L}\{v_o(t)\} = \mathcal{L}\{A v_i(t-\tau)\, u(t-\tau)\} = A e^{-\tau s} V_i(s)$$

where $V_i(s)$ is the Laplace transform of $v_i(t)$.
The transfer function $H(s)$ is given by

$$H(s) = \frac{V_o(s)}{V_i(s)} = A e^{-\tau s}$$

In the frequency domain, $H(j\omega) = A e^{-j\omega\tau}$.
$H(j\omega)$ has a constant amplitude and a phase angle that decreases linearly with frequency.

10-1.3

1. Gain Calculations:
   a. Power into detector: $(160\mu V)^2/9000\Omega = 2.844 \times 10^{-12}$ W.
   b. Power into filter output termination: $(160\ \mu V)^2/700\Omega = 3.657 \times 10^{-11}$ W
   c. Total power out of filter (a+b) $= 3.942 \times 10^{-11}$ W.
   d. Power into filter $= 3.942 \times 10^{-11} \times 10^{6/10} = 1.569 \times 10^{-10}$ W.
   e. Power available from antenna $= (3\mu V)^2/300\Omega = 3.000 \times 10^{-14}$ W.
   f. Required gain $= 10 \log_{10}(d/e) = \underline{37.19\text{ dB}}$.

10-1.3 continued

2. Noise Figure Calculations:

These calculations assume that the thermal noise contributed by the filter and its terminating resistance is negligible. Then the basic requirement becomes one of achieving a 10 dB SNR at the filter input.

a. Signal power present at filter input = 1.569×10^{-10} W.

b. Maximum permissible noise power at filter input for 10 dB SNR = $1.569 \times 10^{-10} / 10^{1.0} = 1.569 \times 10^{-11}$ W.

c. Maximum permissible noise power referred to RF amplifier input = $1.569 \times 10^{-11} \times 10^{-3.719} = 2.997 \times 10^{-15}$ W.

d. Noise power supplied by antenna = kTB = $1.38 \times 10^{-23} \times 300 \times 200 \times 10^{3} = 8.28 \times 10^{-16}$ W.

e. Maximum permissible internal noise (referred to RF amplifier input) = c - d = 2.169×10^{-15} W.

f. Effective input noise temperature of amplifier $T_e = 2.169 \times 10^{-15} / (1.38 \times 10^{-23} \times 200 \times 10^{3}) = 785.87$ K

g. Minimum noise figure = $(T_o + T_e)/T_o = (785.87 + 290)/290$ = 3.71 or 5.69 dB. Note that a standard $T_o = 290$ K was used.

10-2.1

$$\left| \frac{v}{v_{max}} \right| = \frac{1}{\sqrt{1 + Q_t^2 \left(\frac{f}{f_o} - \frac{f_o}{f} \right)^2}} \quad (1)$$

(a) When $f = f_c$, then $|v/v_{max}| = 0.707$. Hence

$$0.707 = \frac{1}{\sqrt{1 + Q_t^2 \left(\frac{f_c}{f_o} - \frac{f_o}{f_c} \right)^2}} \quad \text{and} \quad 0.5 = \frac{1}{1 + Q_t^2 \left(\frac{f_c}{f_o} - \frac{f_o}{f_c} \right)^2}$$

This can be rewritten as $Q_t = -\frac{1}{\frac{f_c}{f_o} - \frac{f_o}{f_c}} = -\left(\frac{f_o f_c}{f_c^2 - f_o^2} \right)$

and solved for f_c to yield

$$f_c = \frac{-f_o + \sqrt{f_o^2 + 4 f_o^2 Q_t^2}}{2 Q_t} = \underline{450.472749 \text{ kHz}}$$

(b) Let $f = f_c + \delta f$; use Eqn. (1). Some results are:

10-2.1 continued

δf, kHz	f, kHz	$\lvert v/v_{max}\rvert$	δf, kHz	f, kHz	$\lvert v/v_{max}\rvert$
+0.5	450.973	0.747	-0.5	449.973	0.669
+1.0	451.473	0.789	-1.0	449.473	0.633
+1.5	451.973	0.832	-1.5	448.973	0.600
+2.0	452.473	0.874	-2.0	448.473	0.569
+2.5	452.973	0.913	-2.5	447.973	0.541
+3.0	453.473	0.948	-3.0	447.473	0.514
+3.5	453.973	0.975	-3.5	446.972	0.490
+4.0	454.473	0.993	-4.0	446.473	0.467
+4.5	454.973	1.000	-4.5	445.973	0.446
+5.0	455.473	0.995	-5.0	445.473	0.427

A plot of $\lvert v/v_{max}\rvert$ versus δf is almost linear between $\pm$ 4 kHz, indicating that slope detection would work with this circuit for narrow band FM.

(c) Tabulate values of $\lvert v/v_{max}\rvert$ for the appropriate range of δf as in part b.

δf, kHz	f, kHz	$\lvert v/v_{max}\rvert$	δf, kHz	f, kHz	$\lvert v/v_{max}\rvert$
+5	455.473	0.995	-5	445.473	0.427
+10	460.473	0.642	-10	440.473	0.294
+20	470.473	0.286	-20	430.473	0.178
+30	480.473	0.180	-30	420.473	0.126
+50	500.473	0.104	-50	400.473	0.078
+75	525.473	0.069	-75	375.473	0.052

A plot of $\lvert v/v_{max}\rvert$ is highly nonlinear. The detected signal would be distorted beyond use.

10-3.1

Waveforms for the limited IF signal, for Q, and for $\bar{Q}$ appear on the next page.

Let τ_0 be half the IF period at the center frequency. The half period at the instaneous frequency is $\tau_0 + \Delta\tau$. The time that $\bar{Q}$ has a non-zero output is

$$t_{\bar{Q}} = 2(\tau_0 + \Delta\tau) - \tau_0 = \tau_0 + 2\Delta\tau.$$

For the greatest positive detector output, $t_{\bar{Q}} = 0$. Call the associated value of $\Delta\tau$ $\Delta\tau_{max}$.

10-3.1 continued

$\tau_o + 2\Delta\tau_{max} = 0$ and $\Delta\tau_{max} = -\frac{\tau_o}{2}$

For the greatest negative detector output $\bar{Q}$ is on for the longest possible part of one IF cycle. Hence $\Delta\tau$ must be adjusted to maximize

$$\frac{\text{time } \bar{Q} \text{ is on}}{\text{time } Q \text{ is on}} = \frac{\tau_o + 2\Delta\tau}{\tau_o} .$$

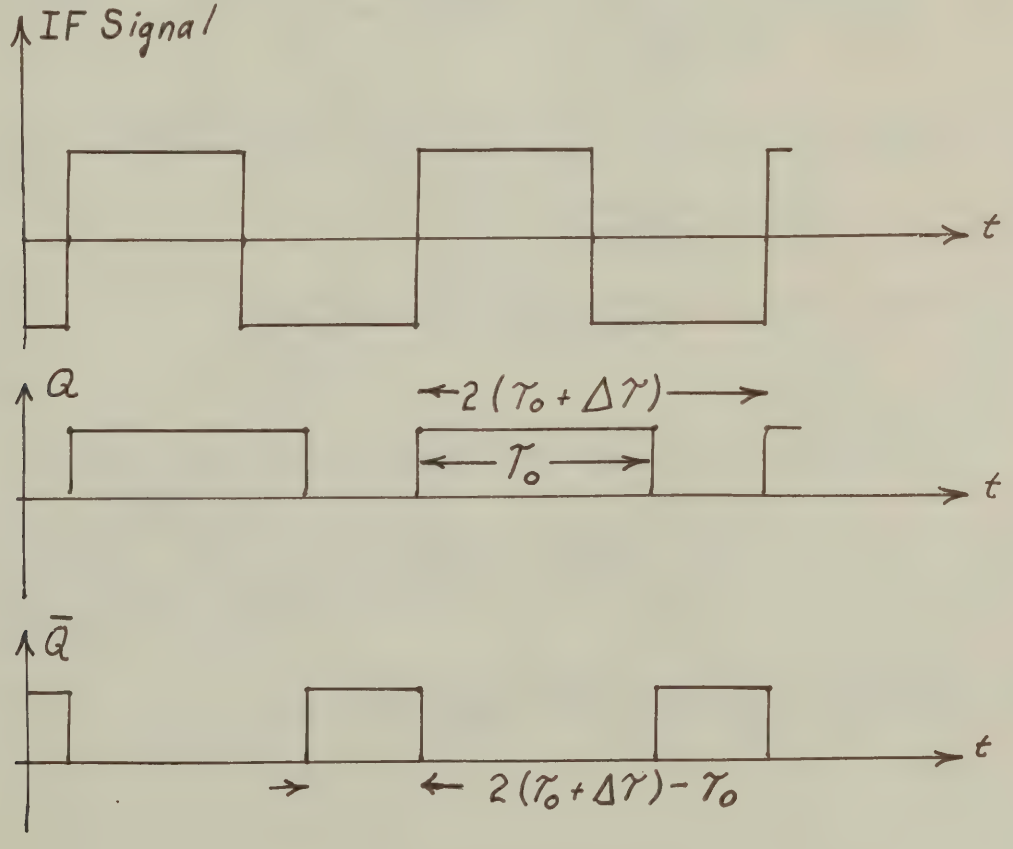
This expression is maximized as $\Delta\tau \to \infty$. Hence the detector would work to dc if the RC integrators could integrate at that low a frequency. In practice of course they can not.

The maximum symmetric period limits are $\tau_o \pm \tau_o/2$. These correspond to frequencies

$$f_{min} = \frac{1}{\tau_o + \tau_o/2} = \frac{2}{3\tau_o} = \underline{\frac{2}{3} F_o} \quad \text{and}$$

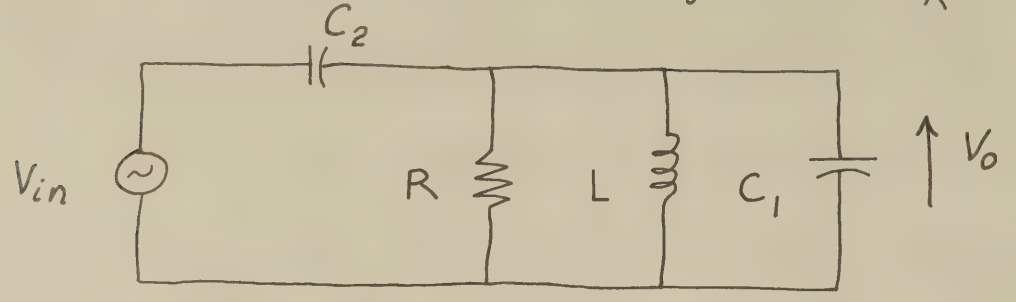
$$f_{max} = \frac{1}{\tau_o - \tau_o/2} = \frac{2}{\tau_o} = \underline{2F_o}$$

Theoretically the detector could work from $f = 0$ to $2F_o$



10-3.2

The circuit is sketched below. In the equations $G = \frac{1}{R}$.



Let $Y_p = 1/Z_p$ be the admittance of R, L, and C_1 in parallel.

$$Y_p = G + j(\omega C_1 - \frac{1}{\omega L}) = G\left[1 + j(\frac{\omega C_1}{G} - \frac{1}{\omega LG})\right]$$

Make the following substitutions.

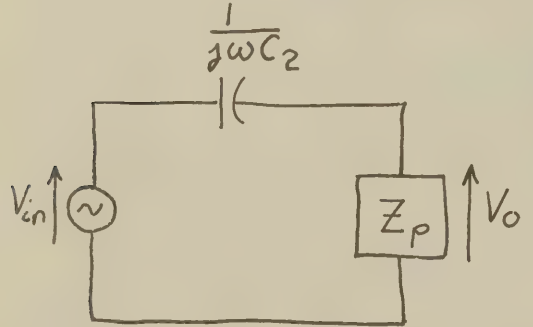
(1) $\frac{\omega C_1}{G} = \frac{\omega}{\omega_0}\frac{\omega_0 C_1}{G} = \frac{\omega}{\omega_0} Q_0$

(2) $\frac{1}{\omega LG} = \frac{1}{\omega \frac{1}{\omega_0^2 C_1} G} = \frac{\omega_0^2 C_1}{\omega G} = \frac{\omega_0}{\omega}\frac{\omega_0 C_1}{G} = \frac{\omega_0}{\omega} Q_0$

Hence $Y_p = G\left[1 + jQ_0(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega})\right] = G(1 + jQ_0\delta)$

and $Z_p = \frac{1}{Y_p} = \frac{R}{(1+jQ_0\delta)}$

The circuit may be viewed as a voltage divider.



$$\frac{V_o}{V_{in}} = \frac{\frac{R}{1+jQ_0\delta}}{\frac{1}{j\omega C_2} + \frac{R}{1+jQ_0\delta}}$$

At resonance $\omega_0 C_2 R = 1$ and $\frac{1}{\omega C_2} = \frac{\omega_0 R}{\omega}$.

10-3.2 continued

$$\text{Hence } \frac{V_o}{V_{in}} = \frac{\frac{R}{1+jQ_o\delta}}{-\frac{j\omega_o R}{\omega} + \frac{R}{1+jQ_o\delta}} = \frac{\frac{1}{1+jQ_o\delta}}{\frac{1}{1+jQ_o\delta} - j\frac{\omega_o}{\omega}}$$

$$= \frac{\frac{1}{1+jQ_o\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)}}{\frac{1}{1+jQ_o\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)} - j\frac{\omega_o}{\omega}} = \frac{1}{1-j\frac{\omega_o}{\omega}\left[1+jQ_o\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)\right]}$$

$$= \frac{1}{1+\frac{Q_o\omega_o}{\omega}\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right) - j\frac{\omega_o}{\omega}}.$$

$$\text{Phase}\left\{\frac{V_o}{V_{in}}\right\} = -\tan^{-1}\left\{\frac{-\frac{\omega_o}{\omega}}{1+\frac{\omega_o}{\omega}Q_o\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)}\right\}$$

$$= \tan^{-1}\left\{\frac{\frac{\omega_o}{\omega}}{1+\frac{\omega_o}{\omega}Q_o\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)}\right\}. \text{ This is Eqn (10-34).}$$

10-3.3

This problem also refers to Eqn. (10-34).

$$\frac{f_o}{\sqrt{1+1/Q_o}} = \frac{10.7\,MHz}{\sqrt{1+1/50}} = 10.5946\,MHz$$

$$f_o - \frac{f_o}{\sqrt{1+1/Q_o}} = 10.7 - 10.5946\,MHz = 0.1054\,MHz.$$

The frequency range of interest is $f_o - 0.1054\,MHz$ to $f_o + 0.1054\,MHz$. This is equivalent to $10.5946 \le F \le 10.8054\,MHz$ or $0.9901 \le \frac{F}{f_o} \le 1.0099$.

Values of $\Delta\theta$ calculated from Eqn. (10-34) appear on the next page.

10-3.3 continued

f/F_o	F, MHz	$\Delta\theta$, deg.	f/F_o	F, MHz	$\Delta\theta$, deg
1.000	10.700	45.00	1.001	10.7107	42.25
0.999	10.6893	48.05	1.002	10.7214	39.76
0.998	10.6786	51.42	1.003	10.7321	37.52
0.997	10.6679	55.14	1.005	10.7535	33.63
0.995	10.6465	63.72	1.008	10.7856	28.91
0.992	10.6144	79.31	1.010	10.8070	26.51
0.991	10.6037	85.03			

These numbers may be displayed in plots of $\Delta\theta$ vs f or $\Delta\theta$ vs $\frac{f}{F_o}$.

10-3.4

(a) $f_{min} = \dfrac{F_o}{\sqrt{1+1/Q_o}} = \dfrac{10.7\,MHz}{\sqrt{1+1/20}} = \underline{10.442\,MHz}$.

(b) $Q_o = \omega_o C_1 R = 31.46$

$f_{min} = \dfrac{10.7\,MHz}{\sqrt{1+1/31.46}} = \underline{10.534\,MHz}$.

10-4.1

$F(t)^+ = \frac{1}{2} + \frac{2}{\pi}\cos(\omega_o t) + \frac{3}{2\pi}\cos(3\omega_o t) - \ldots$

$V_c = (L+R) + P_o \cos\left(\frac{\omega_o t}{2}\right) + (L-R)\cos(\omega_o t)$

$V_c \times F(t)^+ = \frac{L+R}{2} + \frac{2}{\pi}(L-R)\cos^2(\omega_o t) + \text{higher order terms.}$

$\cos^2(\omega_o t) = \dfrac{1+\cos(2\omega_o t)}{2}$

Hence $V_c \times F(t)^+ = \dfrac{L+R}{2} + \dfrac{2}{\pi}(L-R)\left(\dfrac{1+\cos(2\omega_o t)}{2}\right) + \ldots$

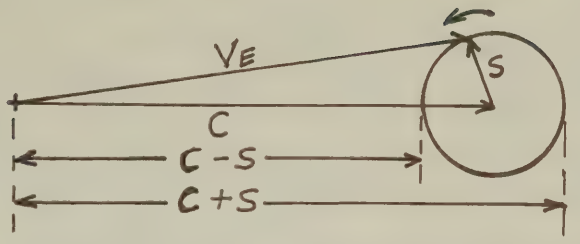
$= (L+R) + \dfrac{1}{\pi}(L-R) + \text{higher frequency terms.}$

CHAPTER 11

P11-4.1 — 11-5.1

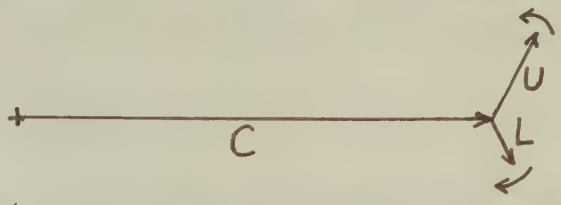
11-4.1

(a) C = carrier
S = upper side frequency
V_E = envelope voltage



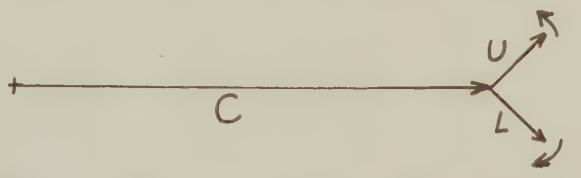
Note: The ratio C/S should be larger than that shown in these figures for undistorted envelope detection.

(b)
Upper side-frequency amplitude $U = \frac{S}{2} + k$.
Lower S.F is $L = \frac{S}{2} - k$.



They have equal, but opposite, angles with respect to C.
Maximum amplitude is $C + (\frac{S}{2} + k) + (\frac{S}{2} - k) = C + S$.
Minimum amplitude is $C - (\frac{S}{2} + k) - (\frac{S}{2} - k) = C - S$
Peak-to-peak envelope variation is $\underline{2S}$ as in part (a).

(c) Both U and L have amplitude S. Their sum lies along the horizontal line.



The total resultant amplitude is C+2S max, C-2S min.
Peak-to-peak envelope variation is $\underline{4S}$.

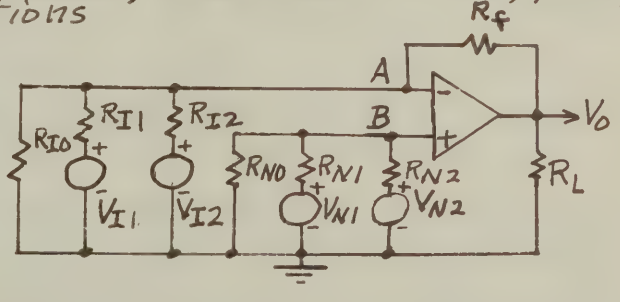
11-5.1

Use ideal op amp theory. Use summing at both inverting (-) and noninverting (+) inputs. (See C. A. Holt, "Electronic Circuits", p. 444, Wiley, 1978.) Write node equations at A and B:

$$\frac{V_o}{R_f} + \frac{V_{I1}}{R_{I1}} + \frac{V_{I2}}{R_{I2}} = \frac{V_A}{R_A}$$

$$\frac{V_{N1}}{R_{N1}} + \frac{V_{N2}}{R_{N2}} = \frac{V_B}{R_B}$$

where $R_A = R_f \| R_{I0} \| R_{I1} \| R_{I2}$
and $R_B = R_{N0} \| R_{N1} \| R_{N2}$



Set $R_A = R_B$ and $V_A = V_B$ to get

$$V_o = R_f \left[\frac{V_{N1}}{R_{N1}} + \frac{V_{N2}}{R_{N2}} - \frac{V_{I1}}{R_{I1}} - \frac{V_{I2}}{R_{I2}} \right]$$

(continued on next page)

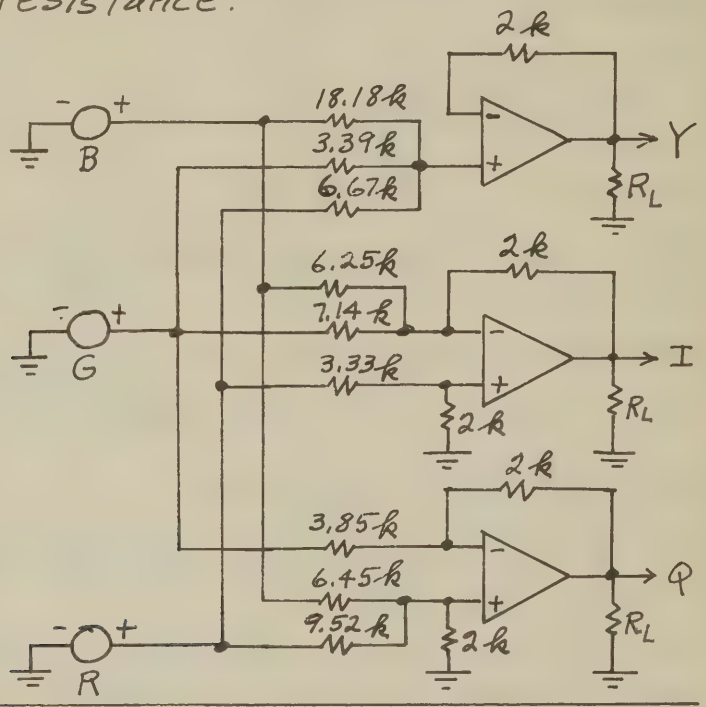
11-5.1 (continued)

For the "I" amplifier, for example: To realize (11-12),
$I = 0.60R - 0.28G - 0.32B$, $R_f = 2k$

$R_{N1} = \frac{2k}{0.60} = 3.33k$, $R_{I1} = \frac{2k}{0.28} = 7.14k$, $R_{I2} = \frac{2k}{0.32} = 6.25k$

$R_{I1} \| R_{I2} \| R_f = 1.25k = R_A$ Let $R_B = R_A = 1.25k = R_{N1} \| R_{NO}$; giving $R_{NO} = 2.0k$. (Also note that $R_{IO} = \infty$.)

Similar calculations for the "Y" and "Q" circuits yield the diagram below. The R, G, and B signal sources are assumed to have zero resistance.



11-6.1

(a) $v_{oQ} = k V_c V_Q [\cos(\omega_c + \omega_Q)t + \cos(\omega_c - \omega_Q)t]$

(b) $v_{oI} = k V_c V_I [\cos(\omega_c t + 90° + \omega_I t) + \cos(\omega_c t + 90° - \omega_I t)]$

(c) The input to the demodulator is:

$$v_i = \underbrace{K V_c \cos(\omega_c t + \phi)}_{(1)} + \underset{(2)}{v_{oQ}} + \underset{(3)}{v_{oI}}$$

For the demodulator, assume $v_o = v_i^2$. Also assume that $K >> k V_Q$ and $k V_I$ so that the product $v_{oQ} v_{oI}$ of terms (2) x (3) will be negligible compared to the products of terms (1) x (2) and (1) x (3).

(continued on next page)

11-6.1 (continued)

When v_i^2 is formed, the product of terms (1)x(2) appears as

$$2kKV_c^2V_Q[\cos(\omega_c t+\phi)\cos(\omega_c t+\omega_Q t)+\cos(\omega_c t+\phi)\cos(\omega_c t-\omega_Q t)]$$

The difference-frequency components of these products are the desired Q output.

$$v_{oQ}' = kKV_c^2V_Q[\cos(\phi-\omega_Q t)+\cos(\phi+\omega_Q t)] \quad (4)$$

$$v_{oQ}' = 2kKV_c^2V_Q\cos\phi\cos\omega_Q t. \quad (5)$$

By a similar procedure the I output can be found.

$$v_{oI}' = kKV_c^2V_I[\cos(\phi-90^\circ-\omega_I t)+\cos(\phi-90^\circ+\omega_I t)]$$

$$v_{oI}' = 2kKV_c^2V_I\cos(\phi-90^\circ)\cos\omega_I t \quad (6)$$

$$v_{oI}' = 2kKV_c^2V_I\sin\phi\cos\omega_I t \quad (7)$$

The 90° phase difference between the Q and I signals is seen from (5) and (7).

Note that $\cos(-\phi)$ could be used in (5) and $\cos(90^\circ-\phi)$ in (7) without altering the values because $\cos x = \cos(-x)$.

PREFACE TO CHAPTERS 12 - 16

The problems for chapters 12 - 16 often represent real design situations adapted from my personal experience. Consequently, the answers are not always simple and direct, and the student may have to make some basic design decisions or judgements, and there may be more than one "right" solution.

For example, a quick glance may.suggest that there is no solution to a problem. The student must then realize that a transformer or matching network can be added to make the load voltage and current compatible with the transistor's ratings. Later in problems dealing with broadband transformers, an exact impedance match may be impossible. The student must then decide whether a match is good enough or whether a more complicated circuit configuration is needed.

While some hints and guidelines are given, most of these design decisions require only common-sense judgements. I hope that such problems will not only teach the students how power amplifiers and transmitters work, but also prepare them for the world of design engineering that many of them will enter shortly after completing this course. I do suggest, however, that the instructor advise the students of the necessity to use common-sense judgement and a little innovation in these problems.

ACKNOWLEDGEMENT

The typing and artwork for Chapters 12 - 16 was done by Patricia L. Scott.

F. H. R.
December 1978

CHAPTER 12 PROBLEMS
P12-1.1 - 12-2.1

12-1.1 A 25 W output requires load voltage and current to be

$$V_{om} = \quad 2 \cdot 50 \cdot 25 = 50 \text{ V}$$

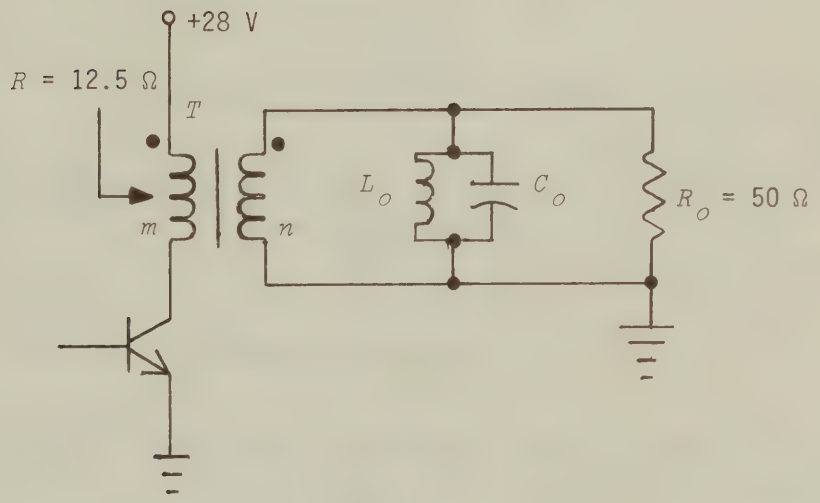
$$I_{om} = 50/50 = 1 \text{ A} .$$

The 28 V supply voltage cannot directly accomodate the peak-to-peak output voltage swing, so a transformer is inserted in the collector circuit as shown. A 2 to 1 voltage reduction by transformer T produces collector voltage swing of $V_{cm} = 50/2 = 25$, hence a peak collector voltage $v_{c,max} = 28 + 25 = 53$ V, 28 V devices are suitable. The peak collector current swing is $1 \cdot 2 = 2$ A, hence the quiescent current is also 2 A and the peak current is $i_{c,max} = 2 + 2 = 4$ A. The dc input power and efficiency are then

$$P_i = 28 \cdot 2 = 56 \text{ W}$$

and

$$\eta = 25/56 = 45\%.$$



At full output power, $P_d = 56 - 25 = 31$ W; however, a safe design will allow for $P_{d,max} = 56$ W that occurs if the RF driving signal is lost.

The 2:1 voltage transformation suggests that $(n,m) = (2,1)$ or $(4,2)$, depending upon the core used. The tuned output circuit should have a Q of about 5, hence at 2 MHz, $X_{L_o} = - X_{C_o} = 50/5 = 10\ \Omega$, therefore $L_o = 0.796\ \mu$H and $C_o = 7{,}958$ pF.

The student should note that this practical design achieves good but not necessarily maximum efficiency for class A operation.

12-2.1 The circuit of Figure 12-5 is used in this problem. The turns ratio of trans-

former T can be found using the load-voltage-and-current approach as in Problem 12-1.1. Load resistance can also be used; to get a 100 W output with a collector voltage swing $V_{cm} \leq 28$ V requires a collector load resistance

$$R \leq (28)^2 \cdot 2 \cdot 100 = 3.92\ \Omega \quad .$$

We then recall that $R = (m/n)^2 R_0$ and start plugging in small integers for m and n. The best choice is $m = 1$, $n = 4$, hence $R = 3.125\ \Omega$; to get R closer to 3.92 Ω requires considerably larger values of m and n. (Remember that $2m + n = 6$ turns must be stuffed through the center of the core of T.)

The selected value of R is now used to calculate the collector voltage and current and other parameters:

$$V_{cm} = \sqrt{(2)(3.125)(100)} = 25\ \text{V}$$

$$I_{cm} = V_{cm}/R = 25/3.125 = 8\ \text{A}$$

$$I_{dc} = 2V_{cm}/\pi = 2(8)/\pi = 5.09\ \text{A}$$

$$P_i = V_{CC}\, I_{dc} = (28)(5.09) = 142.6\ \text{W}.$$

Note that load voltage and current were never actually calculated.

At peak output power,

$$\eta = 100/142.6 = 70.1\%$$

and

$$P_d = 142.6 - 100 = 42.6\ \text{W}.$$

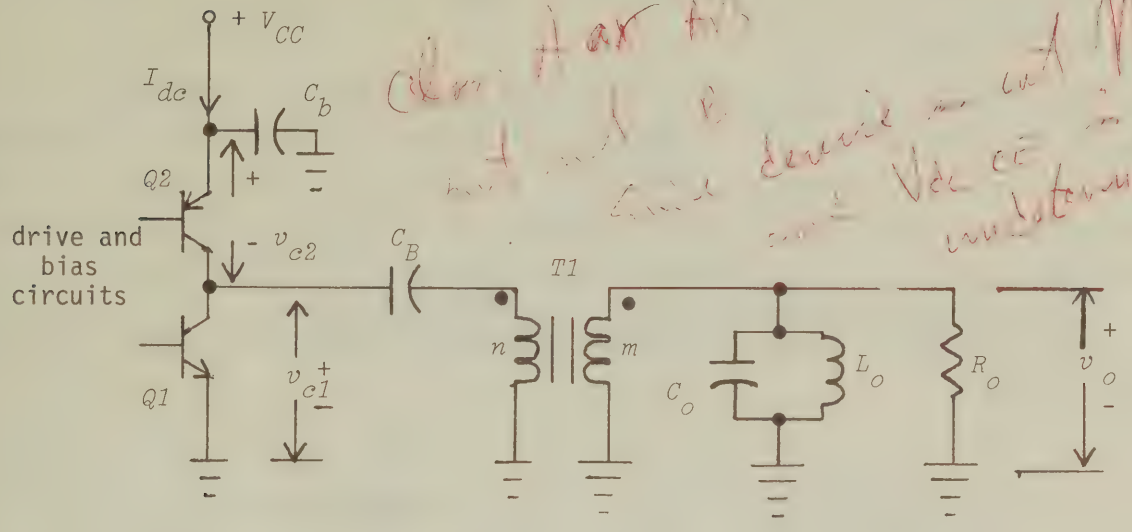
While $P_{d_1} = P_{d_2} = 42.6/2 = 21.3$ W at peak output, the maximum dissipation is calculated from (12-17), hence

$$P_{d_1,max} = P_{d_2,max} = 28^2/\pi^2(3.125) = 25.4\ \text{W}.$$

The heatsink must be capable dissipating $P_{d,max} = 2(25.4) = 50.8$ W. The transistors must have a collector current ratings of $I_{cm} = 8$ A or more and collector voltage ratings of $V_{CC} + V_{cm} = 28 + 25 = 53$ V. The tuned output circuit is the same as that of Problem 12-1.1.

As in Problem 12-1.1, we see that the best practical design has good, but not necessarily maximum, efficiency.

12-2.2 This problem illustrates the design of a complementary PA whose circuit has the form:



First, the ac output voltage is determined from the output power (100 W) and load resistance (50 Ω):

$$V_{om} = \sqrt{2(100)(50)} = 100 \text{ V}.$$

A transformerless circuit would require a peak collector voltage of 200 V, which is presently beyond the ratings of most RF power transistors. A transformer *T1* with $m/n = 2$ gives $V_{cm} = 100/2 = 50$ V. For linear operation, $v_{c1} \geq 0$ and $v_{c2} \leq V_{CC}$. Since the peak-to-peak voltage swing must be $2V_{cm} = 100$ V, $V_{CC} \geq 100$ V; maximum efficiency is obtained with minimum V_{CC}, hence $V_{CC} = 100$ V. These voltages ($v_{c,max} = 100$ V) are consistant with the ratings of moden "50 V" RF power transistors.

The output current is

$$I_{om} = V_{om}/R_o = 100/50 = 2 \text{ A},$$

hence $i_{c,max} = I_{cm} = (m/n)\, I_{om} = 4$ A. The dc input current in a complementary PA is the average of only *one* collector current waveform, hence

$$I_{dc} = I_{cm}/\pi = 4/\pi = 1.27 \text{ A}$$

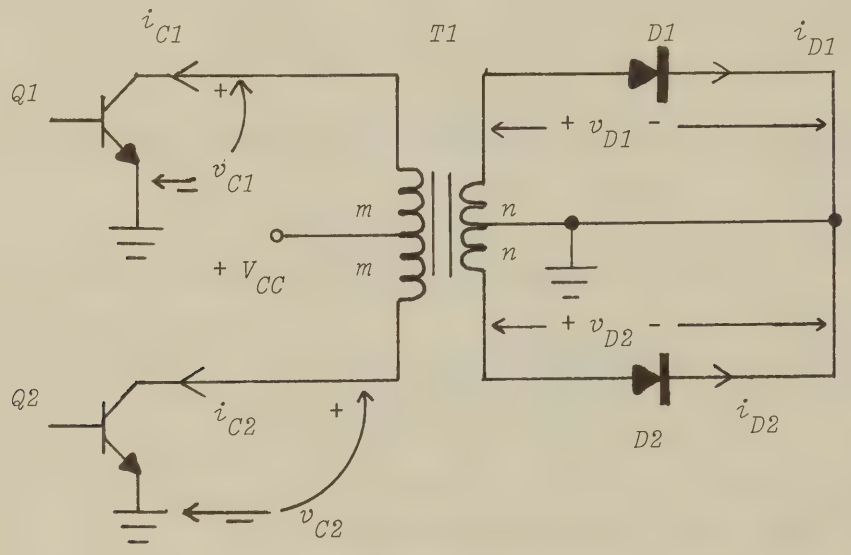
and

$$P_i = V_{CC}\, I_{dc} = 127 \text{ W}.$$

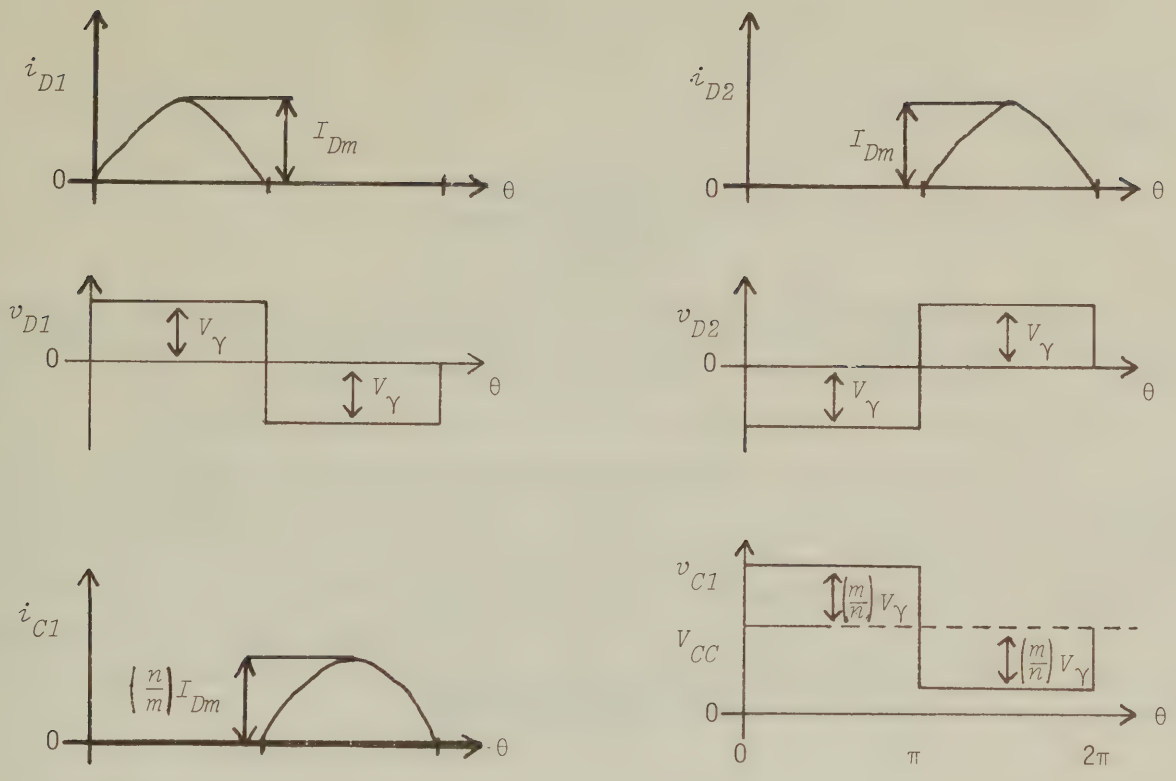
(Output power and load resistance can also be related to V_{CC} through the formula in Appendix 14 - 1.) The efficiency is η = 100/127 = 78.5% as in a push-pull Class B PA. The maximum dissipation is 20.3 W, determined by using (12 - 17) with V_{CC} = 100/2 to be analogous to a push-pull PA.

The values of L_o and C_o are determined as in Problems 12-1.1 and 12-2.1. The reactance of the blocking and bypass capacitors (C_B and C_b) should be small in comparison to the collector load $R = (n/m)^2 R_o$ = 50/4 = 12.5 Ω. For $X_c \leq 12.5/10 = 1.25\ \Omega$, $C \geq 0.063$ μF. Since these values are noncritical, standard values of $C_B = C_b$ = 0.1 μF are selected.

12-2.3 This problem demonstrates that not all PA design problems can be solved by assuming a load resistance. It also forces the student to consider large-signal characteristics of operation.



Diodes *D1* and *D2* represent the bases of a subsequent higher-power PA; when unbiased, they present essentially open circuits to negative current drive and essentially short circuits (with voltage $V_\gamma \simeq 0.7V$) to positive current drive. During the first half of each RF cycle, *D1* conducts positive current and produces a constant voltage $v_{D1} = V_\gamma$; *D2* is cut-off. During the second half of each RF cycle, *D2* conducts positive current and produces a constant voltage $v_{D2} = V_\gamma$; *D1* is cut-off. Reflection by *T1* of the voltage on the conducting diode produces a negative voltage on the cut-off diode, hence v_{D1} and v_{D2} are square waves.



The collector current waveforms are half sinusoids; e.g., $i_{C1}(\theta) = (n/m)\, i_{D2}(\theta)$. The collector voltage waveforms similarly are squarewaves offset by the supply voltage; i.e.,

$$v_{C1}(\theta) = V_{CC} + \left(\frac{m}{n}\right) v_{D1}(\theta) = V_{CC} + \left(\frac{m}{n}\right) V_\gamma s(\theta) \ .$$

Power output is determined by integrating and summing the diode voltage-current products:

$$P_o = 2 \cdot \frac{1}{2\pi} \int_o^\pi (V_\gamma)(I_{Dm} \sin \theta) = \frac{2}{\pi} V_\gamma I_{Dm} \ .$$

Input current is the average of the collector current waveforms, hence $I_{dc} = (2/\pi)(n/m)\, I_{Dm}$, and

$$P_i = \frac{2}{\pi} \left(\frac{n}{m}\right) V_{CC} I_{Dm}$$

The power dissipated in either *Q1* or *Q2* is then

$$P_{d1} = \frac{1}{2}(P_i - P_o) = \frac{1}{\pi} I_{Dm} \left(V_\gamma - \frac{n}{m} V_{CC}\right)$$

and the efficiency is

$$\eta = \frac{P_o}{P_i} = \left(\frac{m}{n}\right) \frac{V_\gamma}{V_{CC}} .$$

It is evident that the efficiency can approach as close to 100 percent as a practical transformer turns ratio (m/n) will allow. Note that the ratio m/n must not produce $(n/m)V_{CC} < V_\gamma$ or the diodes will not be forward-biased during any part of the RF cycle.

12-3.1 The design of power amplifiers for portable communications equipment often requires operation with a supply voltage that can vary considerably with the condition of the batteries. Since the output current in a Class B PA is controlled by the drive, the output is relatively insensitive to the supply voltage (unless saturation occurs). Nonetheless, significant variation in power consumption and dissipation can and do occur.

The first step in designing this PA is to determine a value of R that will accomodate the required drain voltage swing at the minimum supply voltage limit. The upper limit on R can be determined elegantly by converting

$$R \leq \frac{V_{eff}^2}{2 P_o} = \left(\frac{R}{R + R_{on}}\right) \frac{V_{DD}^2}{2 P_o}$$

into a quadratic in R. A suitable value of R can be determined less elegantly but more easily by trial and error as follows: If $R_{on} = 0$, then $R < (24^2)/(2)(16) = 18\ \Omega$. Now $R = 12.5\ \Omega$ is the result of a convenient 4:1 transformer $(m/n = 1/2)$. Assuming $R = 12.5\ \Omega$, use $R_{on} = 2.5\ \Omega$ to calculate

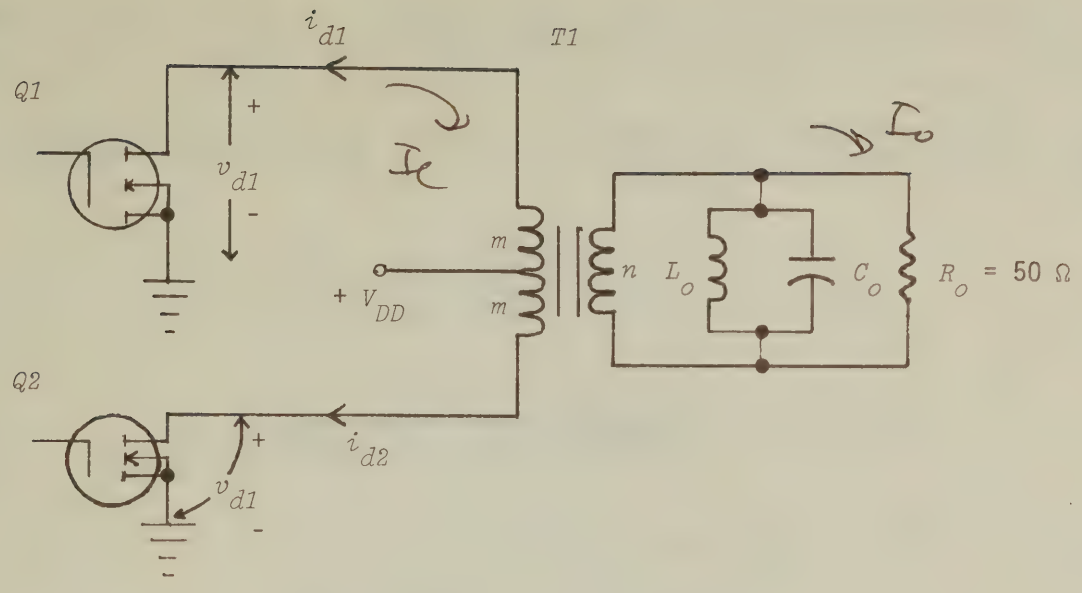
$$20.0 \leq V_{eff} = \frac{12.5}{12.5 + 2.5} V_{DD} < 23.33\ V .$$

With $V_{DD} = 24$,

$$P_o = \frac{V_{eff}^2}{2R} = \frac{20^2}{2 \cdot 12.5} = 16\ W ,$$

hence $R = 12.5\ \Omega$ is OK.

For $Q = 5$ at $f = 27$ MHz, $|X_{Lo}| = |X_{Co}| = 10$, hence $L_o = 58.9$ nH and $C_o = 58.9$ pF.



Now the drain current, input current, and drain voltage swing are functions of the drain load and power output, hence are independent of V_{DD}:

$$I_{dm} = \sqrt{2\ P_o/R} = \sqrt{2 \cdot 16/12.5} = 1.6 \text{ A} \qquad \text{(peak rating)}$$

$$I_{dc} = \frac{2}{\pi} I_{dm} = 1.02 \text{ A}$$

$$V_{dm} = I_{dm} R = 20.0 \text{ V} .$$

The peak current rating is simply the above value (1.6 A) of I_{dm}. The peak voltage rating assumes the maximum supply voltage, hence $V_{D,max} = V_{DD,max} + V_{dm} = 28 + 20 = 48$ V. The remaining characteristics are computed in a straightforward manner:

V_{DD} =	24 V	28 V
P_i	(24)(1.02) = 24.4 W	(28)(1.02) = 28.6 W
η	16/24.4 = 0.655	16/28.56 = 0.560
P_d at P_o = 16 W	24.4 - 16 = 8.43	28.6 - 16 = 12.56 W
$P_{d1,max}$ (use 12-17)	4.67 W	6.35 W
Above occurs with V_{dm} =	15.28 V	17.83 V

The maximum dissipation per device of 6.35 W determines the required heatsink capability. Note the (12 - 17) would not give the maximum dissipation if $2 V_{DD}/\pi < 20$ V = $V_{dm,max}$.

12-3.2 The load factor ρ is convenient in determining the effects of reactive loads. It can be defined either at the collector or load in terms of either impedance or admittance:

$$\rho = \frac{|Z_o|}{R_o} = \frac{1/R_o}{|Y_o|} = \frac{|Z|}{R} = \frac{1/R}{|Y|}$$

$$= \frac{1}{R\sqrt{(1/R)^2 + B^2}} = \frac{1}{\sqrt{1 + (BR)^2}} \quad ,$$

verifying (12 - 22). If the output current is

$$i_o(\theta) = I_{om} \sin \theta = (m/n)\, I_{cm} \sin \theta \ ,$$

then the output voltage is

$$v_o(\theta) = V_{om} \sin (\theta + \phi) \ ,$$

verifying (12 - 21). Also,

$$V_{cm} = (n/m)\, V_{om} = \rho\, R\, I_{cm} \ .$$

Now the input current and power are functions of I_{cm} but not functions of the load, hence (12 - 14) remains valid and

$$P_i = \frac{2}{\pi} V_{CC}\, I_{cm} \ .$$

Since the output voltage produces power only in the output conductance,

$$P_o = \frac{V_{om}^2}{2R_o} = \frac{V_{cm}^2}{2R} = \frac{(\rho\, I_{cm}\, R)^2}{2R} = \frac{\rho^2\, R}{2} I_{cm}^2 \ .$$

The power dissipated in signal device is then

$$P_{d1} = \frac{1}{2}(P_i - P_o) = \frac{1}{\pi} V_{CC}\, I_{cm} - \frac{\rho^2\, R}{4} I_{cm}^2$$

and

$$\frac{\partial P_{d1}}{\partial I_{cm}} = \frac{V_{CC}}{\pi} - \frac{\rho^2 R}{2} I_{cm} .$$

Setting this equal to zero gives

$$I_{cm,max\ dis} = \frac{2 V_{CC}}{\pi \rho^2 R} .$$

Substituting this into the above expressions for P_{d1} and V_{cm} yields

$$P_{d1,max} = \frac{V_{CC}}{\pi}\left(\frac{2 V_{CC}}{\pi \rho^2 R}\right) - \frac{\rho^2 R}{4}\left(\frac{4 V_{CC}^2}{\pi^2 \rho^4 R^2}\right) = \frac{V_{CC}^2}{\pi^2 R}$$

and

$$V_{cm,max\ dis} = \rho\left(\frac{2 V_{CC}}{\pi \rho^2 R}\right) R = \frac{2 V_{CC}}{\pi \rho} ,$$

verifying (12 - 23) and (12 - 24), respectively.

12-3.3 The designer has presumably produced a 100 W PA that works satisfactorily through the efforts associated with problem 12-2.1. The allowable load conditions must now be determined, along with the worst-case performance for power consumption and dissipation requirements.

Saturation Voltage

The design required V_{cm} = 25 V for P_o = 100 W into R_o = 50 Ω. With V_{sat} = 1 V, $V_{cm} \leq$ 27 V for linear operation, hence operation into a 50 Ω load is as predicted in the original design.

Maximum Load Resistance

As load resistance increases, V_{cm} must increase to maintain P_o = 100 W, hence $V_{cm} = V_{eff} = V_{CC} - V_{sat} = 27$ V determines maximum resistance:

$$100 = \frac{27^2}{2 R_{max}}$$

$$R_{max} = 3.65\ \Omega$$

$$R_{o,max} = 16 R_{max} = 58.3\ \Omega .$$

With $R = R_{max}$,

$$I_{cm} = \frac{27}{3.65} = 7.41 \text{ A}$$

$$I_{dc} = \frac{2}{\pi} I_{cm} = 4.72 \text{ A}$$

$$P_i = 28 \cdot 4.72 = 132 \text{ W}$$

$$\eta = 100/132 = 75.7\%$$

Minimum Load Resistance

As the load resistance decreases, the load current must increase to maintain $P_o = 100$ W, hence the 9 A collector current rating determines the minimum resistance:

$$100 = \frac{9^2 R_{min}}{2}$$

$$R_{min} = 2.47 \ \Omega$$

$$R_{o,min} = 16 R_{min} = 39.5 \ \Omega \ .$$

With $R = R_{min}$,

$$I_{dc} = \frac{2}{\pi} \cdot 9 = 5.73 \text{ A}$$

$$P_i = 28 \cdot 5.73 = 160.4 \text{ W}$$

$$\eta = 100/160.4 = 62.3\%$$

$$V_{cm} = 2.47 \cdot 9 = 22.3 \text{ V} \ .$$

Maximum Series Reactance

An output $P_o = 100$ W requires that $I_{om} = 8$ A, with or without series reactance. The maximum series reactance (X_o or X) is therefore determined by the limit $V_{cm} \leq 27$ V for linear operation:

$$8 \ |Z| \leq 27$$

$$\sqrt{R^2 + X^2} = \sqrt{(3.125)^2 + X^2} = |Z| \leq 3.38$$

$$|X| \leq 1.27$$

$$|X_o| \leq 16\ |X| = 20.4\ \Omega\ .$$

Since I_{cm} is unchanged by the inclusion of X_o, performance is also unchanged (from that with $Z_o = 50\ \Omega$). Stability and second-breakdown stress may, however, be affected.

Minimum Shunt Reactance

As a reactance shunting the load decreases, an increasing amount of current is diverted around the load, hence the collector current must increase to maintain $P_o = 100$ W. The 9 A collector current rating therefore determines X_{min} or $X_{o,min}$. Since $P_o = 100$ W requires $V_{om} = 25$ V with or without shunt reactance,

$$|Y| \leq \frac{9}{25} = 0.360 \text{ mho}$$

$$\sqrt{B^2 + (1.50)^2} \leq 0.360/16 = 0.0225$$

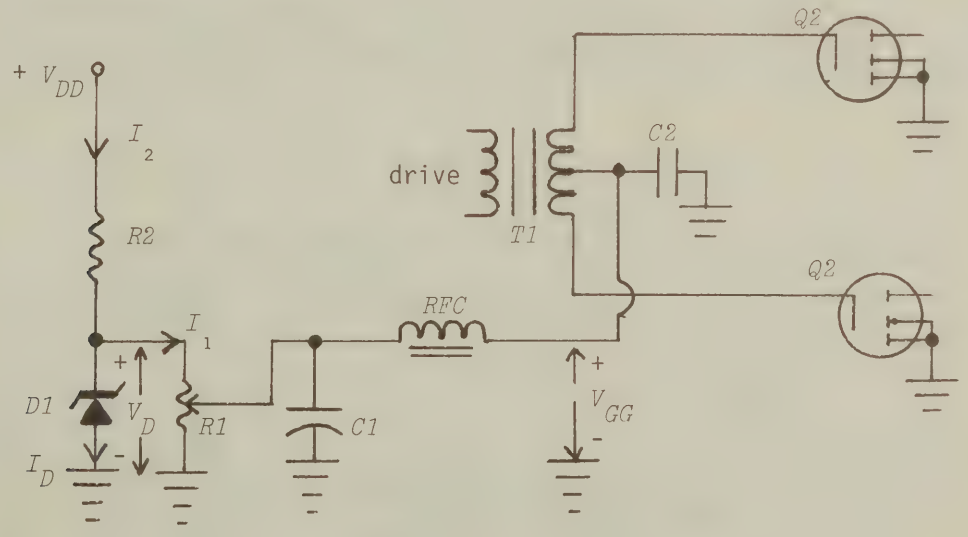
or

$$B_o \leq 0.0103$$

$$X_o \leq \frac{1}{B_o} = 97.01\ \Omega\ .$$

The performance is the same as with $R = R_{min}$.

12-4.1 This is a simple circuit design problem involving providing proper dc and ac current paths. The bias and gate portion of the PA circuit might have the form



The FETs can be regarded as open circuits for dc and it is a good idea to keep the current in *R1* less than 10 percent of that in *D1*, hence

$$R1 = \frac{V_D}{I_D/10} = \frac{3.3\ \text{V}}{1\ \text{mA}} = 3.3\ \text{k}\ \Omega \quad ;$$

$R1$ is adjusted initially for $V_{GG} \simeq 2$ V and then for minimum IMD. Since

$$I_2 = I_D + I_1 = 11\ \text{mA}$$

must be provided at $V_{DD} = 24$,

$$R2 \leq 24/0.011 = 1882\ \Omega$$

and we select the convenient value

$$R2 = 1.5\ \text{k}\ \Omega \quad .$$

While zener diode regulation is not impaired by $V_{DD} = 28$, maximum power consumption must be calculated for that V_{DD}, thus

$$I_{2,max} = \frac{28 - 3.3}{1500} = 16.5\ \text{mA}$$

and

$$P_{bias,max} = 28 \cdot 0.0165 = 461\ \text{mW} \ .$$

Most of this is consumed in $R2$.

Capacitor $C1$ should have a reactance considerably lower than that of resistor $R1$, hence for $X_{C1} \leq 330$ ohms at $f = 27$ MHz, $C1 \geq 17.9$ pF and $C1 = 0.001\ \mu\text{F} + 100$ pF is selected. The ac gate impedance was not specified but can be assumed to be high; $C2 = 0.001\ \mu\text{F} + 100$ pF and $RFC = 20\ \mu\text{H}$ ($X_{RFC} = 3393$ ohms) should be OK.

12-5.1 This problem involves using ac and dc circuit characteristics and making the right gross assumptions about transistor characteristics. First, rework (12 - 27) to get

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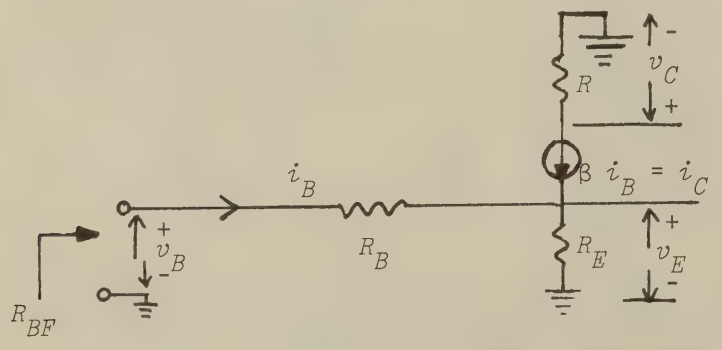
$$v_E = (i_C + i_B)R_E = (\beta\ i_B + i_B)R_E = (\beta + 1)i_B\ R_E \ .$$

Now for ac signals, V_γ is ignored and the equivalent circuit is:



Therefore, using the above equation and diagram

$$v_B = v_E + i_B R_B = [(\beta + 1)R_E + R_B]\, i_B$$

and

$$R_{BF} = \frac{V_B}{i_B} = (\beta + 1)R_E + R_B \ ,$$

which is (12 - 28).

The voltage gain G_v is then

$$G_v = \frac{v_C}{v_B} = \frac{i_C R}{v_B}$$

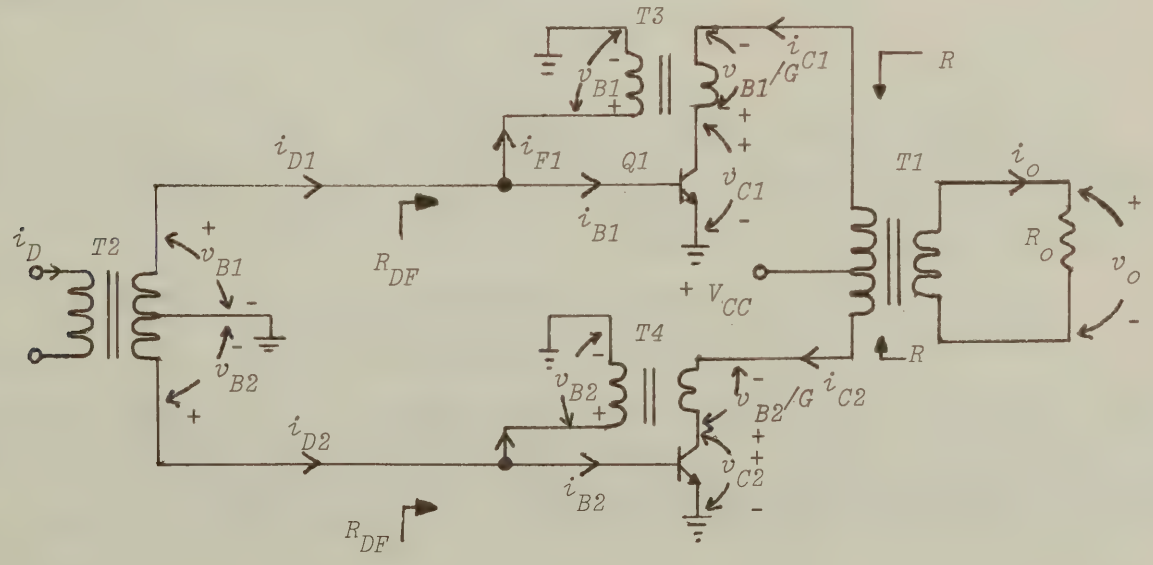
$$= \frac{\beta\, i_B R}{[(\beta + 1)R_E + R_B]\, i_B} = \frac{R}{\left(\frac{\beta + 1}{\beta}\right)R_E + R_B/\beta} \ .$$

As $\beta \to \infty$, $(\beta + 1)/\beta \to 1$ and $R_B/\beta \to 0$, hence

$$G_v \simeq \frac{R}{R_E} \ ,$$

indicating independence from variations in both β and R_B. Note that neither β nor R_B was ever assumed to be constant in the above derivations.

12-5.2 There are many ways of accomplishing feedback in a PA besides the collector and emitter feedback discussed in the text. A simplified current-to-current feedback circuit is shown below.



If the turns-ratio of $T3$ (and $T4$) is G, then the feedback current i_{F1} is

$$i_{F1} = \frac{i_{C1}}{G} \quad .$$

Since the drive current i_D is determined by the driving amplifier,

$$\frac{1}{\beta} i_{C1} = i_{B1} = i_{D1} - i_{F1} = i_{D1} - \frac{i_{C1}}{G}$$

and as $\beta \to \infty$

$$i_{C1} \simeq G\, i_{D1} \ ,$$

hence G is the amplifier's current gain. (A similar relationship applies to the currents associated with $Q2$).

The collector voltage is

$$V_{cm} = I_{cm} R - \frac{V_{b1m}}{G} \quad ,$$

and the decrease by V_{b1m}/G indicates a small decrease in efficiency associated with dumping some of the output power into the base circuit.

The input (driving) impedance can be found from

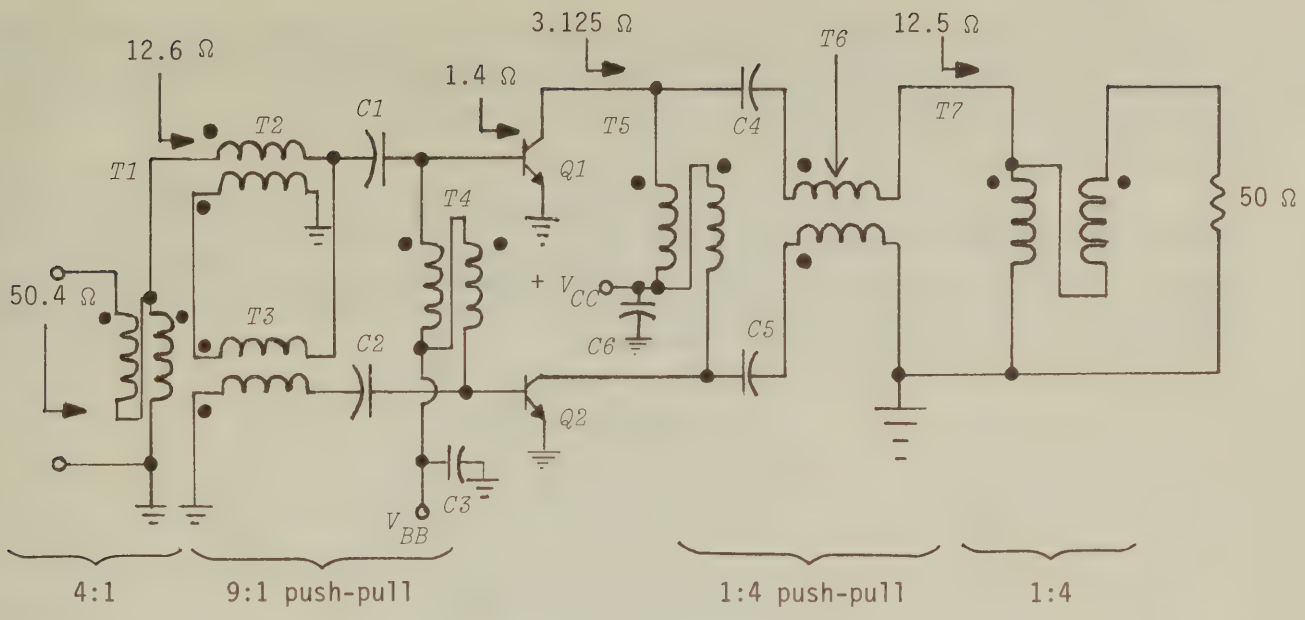
$$R_{DF} = \frac{V_{b1m}}{I_{d1m}} = \frac{I_{b1m} R_B}{I_{d1m}}$$

$$= \frac{I_{c1} R_B/\beta}{I_{c1}/G} = \frac{G}{\beta} R_B \ .$$

Advantages of this feedback arrangement include: (1) no power is wasted in a feedback resistor, and (2) a series of current-to-current linearized amplifiers can be formed with minimal consideration of input-impedance variation. A disadvantage is that $T3$ and $T4$ may be difficult to implement in a broadband configuration or may degrade the broadband characteristics of $T1$ and $T2$.

The reader should note that the connection of a tertiary winding on $T1$ to a tertiary winding on $T2$ can produce voltage-to-voltage linearization.

12-6.1 There are probably several acceptable solutions to this problem; this one appears to be the most practical. Begin with the requirement from problem 12-2.1 that $R = 3.125\ \Omega$ at the collectors. The 16:1 transformation for push-pull operation requires at least three transformers ($T5$, $T6$, and $T7$):



A push-pull configuration similar to that of Figure 12 - 13 (c) would require 4 transformers, as would putting a 1:4 transformer at each collector and using the hybrid-balun to convert 50 Ω to 12.5 Ω (push=pull).

Now consider the input: The specified R_B = 1.4 Ω requires a transformation of 50/1.4 = 35.7. Fortunately, 35.7 ≃ 36 = 4•9 = $2^2 \cdot 3^2$, suggesting a broadband configuration will be quite close. The 9:1 transformation is accomplished by *T2*, *T3*, and *T4* using the push-pull configuration of Figure 12 - 13 (c). The 4:1 transformation is then accomplished by a single transformer *T1*. The resultant 50.4 Ω driving impedance is quite satisfactory and attempts to add more transformers to affect a 1 percent improvement is not justified. (In fact, additional transformers would probably significantly degrade broadband performance.)

The data in Figures 12 - 12 (b), 12 - 13 (b), and 12 - 13 (c) is now used to calculate the characteristic impedances of the lines:

Transformer	Z_0	Comment
T1	(2)(12.6) = 25.2 Ω	25 Ω is close enough, can connect two 50 Ω lines in parallel.
T2, *T4*	(1/3)(12.6) = 4.2 Ω	
T3	(2/3)(12.6) = 8.4 Ω	
T5	(2)(3.125) = 6.25 Ω	
T6	(1)(12.5) = 12.5 Ω	
T7	(2)(12.5) = 25 Ω	

In a broadband design, it is especially important to keep stray reactances to a minimum; therefore the reactances of the capacitors should be 1 percent or less of these

the adjacent circuit. Thus $X_{C1} < 0.014\ \Omega$ at 2 MHz, hence $C1 > 5.68\ \mu F$; a parallel combination of 10 μF, 0.1 μF, 0.001 μF and 10 pF would therefore be used for *C1*, *C2*, and *C3*. Similarly, $X_{C4} < 0.03125\ \Omega$, hence $C4 > 2.55\ \mu F$ and combination of 4.7 μF, 0.05 μF, 470 pF and 10 pF might be used for *C4*, *C5*, and *C6*.

Now for *T5*; in Problem 12-3.3 we determined that specified performance within specified transistor ratings required that $|X_{o,series}| \leq 20.4\ \Omega$ and $|X_{o,shunt}| \geq 97\ \Omega$. Since *T6* acts as a balum, its effects are small in comparison to these of *T5* and *T7* and the series and shunt reactance effects can be split equally between *T5* and *T7* for design. The minimum shunt reactance of *T5* is therefore

$$X_M \geq 2(97/16) = 12.13\ \Omega$$

At $f = 2$ MHz,

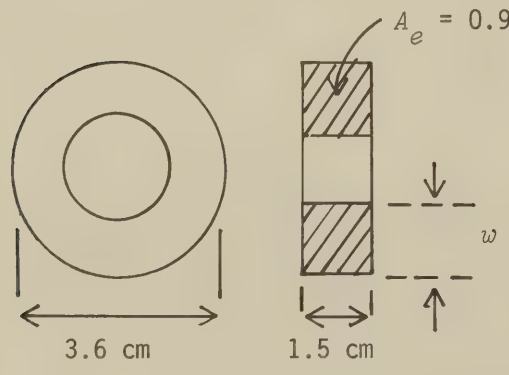
$$L_M \geq \frac{12.13}{2\pi f} = 0.965\ \mu H.$$

Noting that $A_L = 3639\ mH/(1000\ turns)^2 = 3.639\ \mu H/(turn)^2$, it is evident that $n = 1$ turn will provide adequate shunt reactance is. To keep the flux level under 3000/10 = 300 gauss, use (12 - 24) with $V_{rms} = 27/\sqrt{2} = 19.09$ V:

$$B = \frac{19.09 \cdot 10^8}{4.44 \cdot 2 \cdot 10^6 \cdot 0.977 \cdot n} = \frac{220}{n} \quad ,$$

hence $n = 1$ also produces a satisfactory maximum flux.

To evaluate the high-frequency response, first determine the length of 1 turn.



Since $1.5\ w = 0.977$ cm, $w = 0.65$ cm and one turn must be at least $2(1.5 + 0.65) = 4.3$ cm long; with insulation and spiral one turn might be as long as 5 cm.

At $f = 30$ MHz, $\lambda = 10$ m in free-space and in a transmission-line with $v_p = 0.9$ c, $\lambda = 9$ m. Since 9 m/10 >> 5 cm, we expect no problem with the high-frequency response. I would suggest using $n = 3$ to center transformer response in the desired range of operations.

12-7.1 The only trick to this problem is remembering that R is twice as large as it would be in a push-pull PA output transformer. Begin with the voltage V_1:

$$V_1 = V_h + (m/n)V_o$$

$$= (I_1 - I_2)R_h + (m/n)R_o\, I_o$$

Now use (12 - 37):

$$V_1 = (I_1 - I_2)R_h + (m/n)^2 R_o\, (I_1 + I_2)$$

$$= (I_1 - I_2)R_h + (I_1 + I_2)(R/2)$$

$$= (R/2 + R_h)I_1 + (R/2 - R_h)I_2 \ .$$

Now it is evident that if $R_h = R/2$, all dependence upon I_2 dissappears and

$$V_1 = I_1\, R$$

hence

$$R_1 = R \ .$$

12-7.2 Be careful to notethat in Figure 12 - 15b, $V_2 = -\, I_2\, R_2$. First, determine the voltage on R_h by equating the voltages across the two n-turn windings:

$$V_1 - V_h = V_h - V_2$$

or

$$V_h = \tfrac{1}{2}\,(V_1 + V_2) = \tfrac{1}{2}\,(I_1\, R_1 - I_2\, R_2).$$

Now the current in R_h is

$$I_2 - I_1 = \frac{V_h}{R_h} = \frac{I_1\, R_1 - I_2\, R_2}{2\, R_h} \ ,$$

hence

$$I_1\,(2\, R_h + R_1) = I_2\,(2\, R_h + R_2) \ .$$

Using (12 - 37),

$$I_1 + I_2 = \frac{m}{n} I \ ,$$

and using the above relationship between I_1 and I_2,

$$I_1 \left(1 + \frac{2 R_h + R_1}{2 R_h + R_2}\right) = \frac{m}{n} I$$

hence

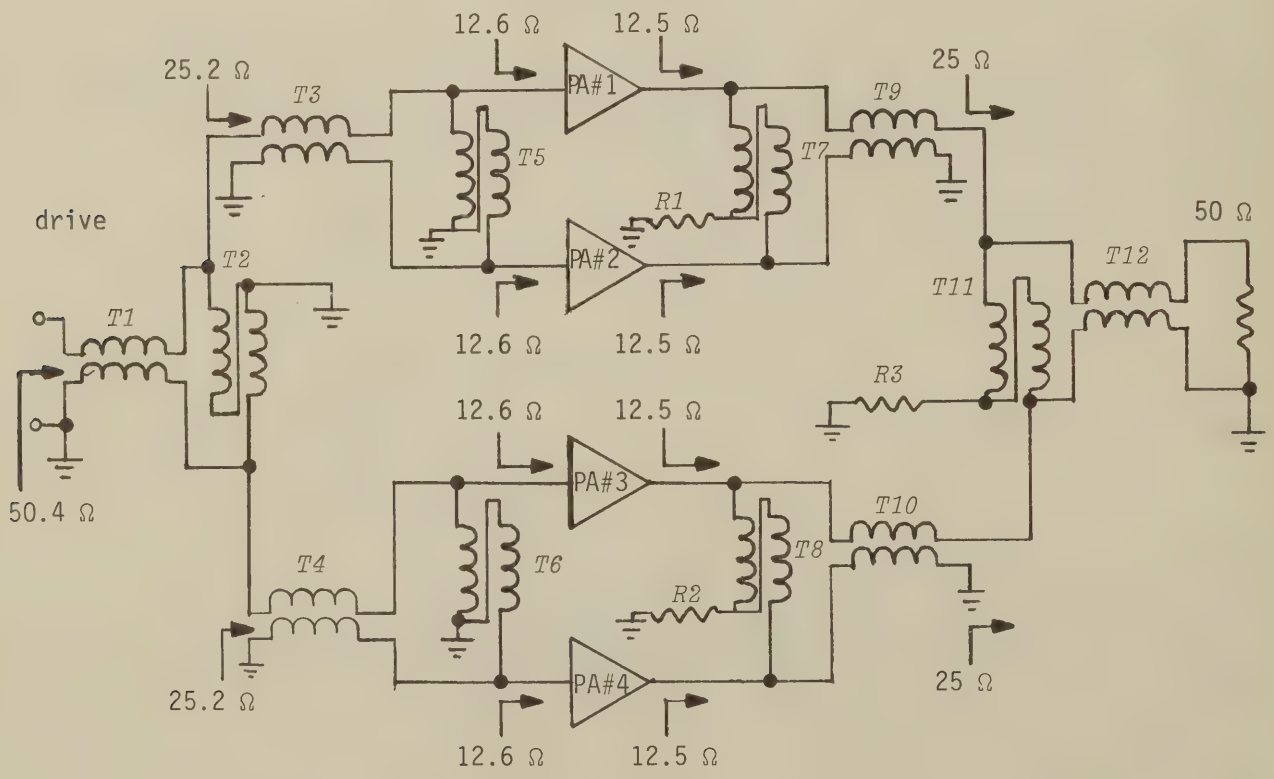
$$I_1 \left(\frac{4 R_h + R_1 + R_2}{2 R_h + R_2}\right) = \frac{m}{n} I$$

or

$$\frac{I_1}{I} = \frac{m}{n} \frac{2 R_h + R_2}{4 R_h + R_1 + R_2} \ ,$$

which is (12 - 38).

12-7.3 Since four PAs must be combined, drive power is first split in half and then each half is split in half to provide four equal driving powers. Two pair of 100 W outputs are similarly first combined to produce two 200 W outputs, which are then combined into a single 400 W output. Using the configuration of Figure 12 - 15 c for all splitting and combining of power, we get the following arrangement:



While establishing the splitting and combining network configurations, we discover that a double 2:1 impedance conversion is accomplished, eliminating the need for the first and last transformers (*T1* and *T7*) called for in Problem 12 - 6; the modified PAs then have 12.6 and 12.5 Ω input and output impedances to match those of the splitting and combining networks.

In accordance with the discussions in the text and the result of Problem 12-7.1, the values of resisters *R1*, *R2*, and *R3* are half the impedance seen by either input.

Thus $R1 = R2 = 12.5/2 = 6.25\ \Omega$

and $R3 = 25/2 = 12.5\ \Omega.$

Inspection of Figure 12 - 15c shows that the balun transformers are subject to the same voltages and currents as are their respective load, hence their characteristic impedances are the load impedances:

$T1, T12 : 50\ \Omega$

$T3, T4, T9, T10 : 25\ \Omega\ .$

Inspection of that figure also shows that with the proper values of *R1*, *R2*, and *R3* and equal PA input and output impedances, no current flows in the hybrid transformers the their characteristic impedance therefore appears indeterminant. A reasonable value can be selected by assuming failure of one of the two inputs or outputs (i.e., open circuit in place of one PA in the pair). This produces

$T2, T11 : 25\ \Omega$

$T3, T4, T9, T10 : 12.5\ \Omega\ .$

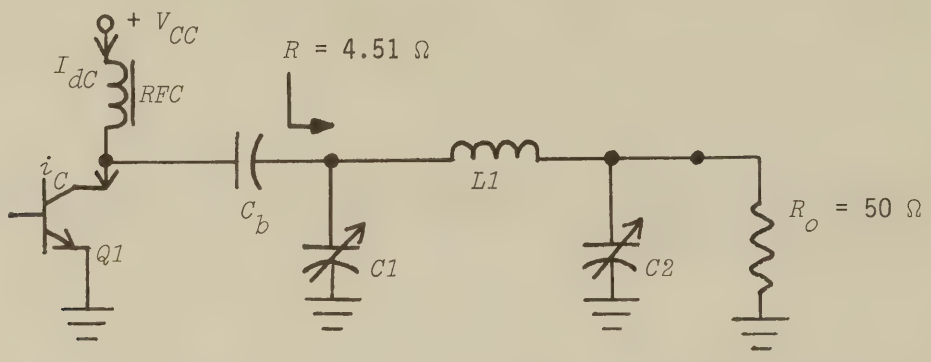
These values should be noncritical to normal circuit operation and the author would make sure the reactances of magnetizing inductances are considerably larger then the nearby circuit impedances.

CHAPTER 13 PROBLEMS
P13-1.1

13-1.1 First, since $V_{eff} = V_{CC} - V_{sat} = 28 - 2 = 26$ V, the collector load resistance must be

$$R = \frac{26^2}{2 \cdot 75} = 4.51\ \Omega \quad .$$

This would be obtained with a Π-network and the circuit would have the form



At f = 50 MHz, let $X_{RFC} \geq 50\ R$, hence $RFC \geq 0.72$ μH. Similarly, let $X_{C_b} \leq R/50$, hence $C_b \geq 0.035$ μF. To be safe, use $RFC \cong 1$ μH and $C_b = 0.05$ μF + 0.001 μF + 100 pF. The matching network can be designed by several methods. Using the formulas in [26] with $Q = 5$,

$$X_{C1} = R/Q = 4.51/5 = 0.9\ \Omega$$

$$C1 = 3529\ \text{pF}$$

$$X_{C2} = R_o\sqrt{\frac{R/R_o}{(Q^2 + 1) - (R/R_o)}} = 50\sqrt{\frac{4.51/50}{(5^2 + 1) - (4.51/50)}}$$

$$= 2.94\ \Omega$$

$$C2 = 1083\ \text{pF}$$

$$X_{L1} = \frac{Q\,R + (R\,R_o/X_{C2})}{Q^2 + 1} = \frac{5 \cdot 4.51 + (4.51 \cdot 50/2.94)}{5^2 + 1}$$

$$= 3.82\ \Omega$$

$$L1 = 12.1\ \text{nH}$$

You may observe that $C1$ and $C2$ seem rather large and $L1$ seems rather small; this illustrates the dificulty of using a Π-network (hence the classical Class C PA concept) with VHF PAs.

The collector voltage swing will equal V_{eff}, hence $v_{C,max} = 28 + 26 = 54$ V. For maximum efficiency, let $i_{C,max} = 15$ A (the rating). Using this, $P_o = 75$ W, and V_{eff} in (12-8) produces

$$P_{max} = \frac{75}{15(26 + 26)} = 0.0962 \quad .$$

Now use either (13-8) iteratively or Figure 13-2 to convert P_{max} into the conduction half angle

$$y = 58.7° = 1.025 \text{ radians} .$$

Next note that

$$I_{CQ} + I_{CD} = 15$$

Using (13-3),

$$I_{CD}(1 - \cos y) = 15$$

$$I_{CD} = 15/0.048 = 31.2\ \text{A}$$

$$I_{CQ} = 15 - 31.2 = -16.2\ \text{A}$$

Using (13-4),

$$I_{dc} = \frac{I_{CD}}{\pi} (\sin y - y \cos y) = 3.2\ \text{A}$$

hence

$$P_i = 3.2\ (28) = 89.6\ \text{W}$$

and $$\eta = 75/89.6 = 83.7\% \ .$$

Note that for $V_{sat} = 0$ with $y = 58.7°$, (13-7) gives $\eta_{ideal} = 90.2\%$, multipication by $V_{eff}/V_{CC} = 0.929$ yields 83.7%.

13-1.2 The circuit has the same form as that of the previous problem. Inspection of Figure 13-2 and iteration using (13-8) produce a maximum value of $P_{max} = 0.1314$ at $y = 122.6° = 2.140$ radians, which implies Class AB operation for maximum output. Using the procedure of the previous problem,

$$I_{CD}(1 - \cos 122.6°) = 15$$

$$I_{CD} = 9.75 \text{ A}$$

$$I_{CQ} = 15 - 9.75 = 5.25 \text{ A}$$

$$I_{dc} = 6.17 \text{ A}$$

$$P_i = 172.8 \text{ W}$$

Now rearrange (13-6) to determine the fundamental-frequency component of collector current

$$I_{cm1} = \frac{V_{cm}}{R} = \frac{I_{CD}}{2\pi}(2y - \sin 2y) = 8.05 \text{ A}.$$

For maximum efficiency within the maximum output constraint, $V_{cm} = V_{eff} = 26$ V, hence

$$R = \frac{V_{cm}}{I_{cm1}} = \frac{26}{8.05} = 3.23 \ \Omega$$

The output power is then

$$P_o = \frac{V_{eff}^2}{2R} = \frac{26^2}{2 \cdot 3.23} = 104.6 \text{ W} \ .$$

hence

$$\eta = 104.6/172.8 = 60.5\% \ .$$

The matching network can be designed as in the previous problem, producing

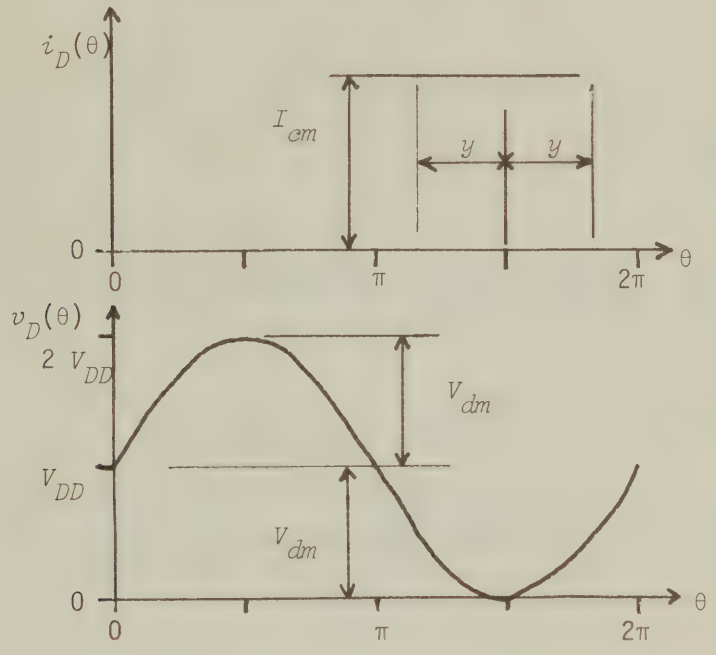
X_{C1} = 0.646 Ω $C1$ = 4863 pF

X_{C2} = 2.49 Ω $C2$ = 1259 pF

X_{L1} = 3.12 Ω $L1$ = 97.9 nH

The values of RFC and C_b are not critical and can be the same as in Problem 13-1.1.

13-1.3 The circuit for this problem is that of Figure 13-1a. The collector waveforms are



First, use integration, tables, or (14-36) to obtain the fundamental-frequency and dc components of $i_D(\theta)$:

$$I_{dm1} = \frac{2}{\pi} I_{dm} \sin y$$

$$I_{dc} = \frac{y}{\pi} I_{dm} .$$

For maximum output and efficiency, $V_{dm} = V_{DD}$, hence

$$\frac{V_{dm}}{R} = \frac{V_{DD}}{R} = I_{dm1} \quad ,$$

$$I_{dm} = \frac{\pi\ V_{DD}}{2\ R\ \sin\ y} \quad ,$$

and

$$I_{dc} = \frac{y}{2\ \sin\ y}\ \frac{V_{DD}}{R} \quad .$$

As usual, $P_i = V_{DD}\ I_{dc}$ and $P_o = V_{DD}^2/2\ R$, hence

$$\eta = \frac{P_o}{P_i} = \frac{V_{DD}^2/2\ R}{V_{DD}^2\ y/2\ R\ \sin\ y} = \frac{\sin\ y}{y} \quad .$$

and

$$P_{max} = \frac{P_o}{v_{d,max}\ i_{d,max}} = \frac{V_{DD}^2/2\ R}{(2\ V_{DD})(\pi V_{DD}/2\ R\ \sin\ y)} = \frac{\sin\ y}{2\ \pi} \quad .$$

y	0	$\pi/2$	π
η	1	$2/\pi$	0
P_{max}	0	$1/2\pi$	0

In general, rectangular current produces lower efficiency but higher output capability than does sinusoidal current. For the PA of example 13-1.1, $\eta = 0.85$ requires

$$y = 0.97 \text{ radians} = 55.6° \quad ,$$

obtained by iteration with the above-derived equation for η. With $P_o = 25$, $\eta = 0.85$ implies

$$P_i = 25/0.85 = 29.4 \text{ W},$$

hence

$$I_{dc} = P_i/V_{DD} = 29.4/12 = 2.45 \text{ A} .$$

Using the second equation derived above,

$$I_{dm} = (\pi/y)\, I_{dc} = 7.93 \text{ A},$$

hence

$$I_{dm1} = (2/\pi)(7.93)(\sin 55.6°) = 4.17 \text{ A}$$

and

$$R = (V_{DD}/I_{dm1}) = 12/4.17 = 2.88\ \Omega .$$

13-2.1 The solution of this problem is a tedious but straightforward exercise in trigonometry. This method of solution can be applied to a variety of problems in which the transistors or FETs in a PA circuit pass through all three regions of operation during each RF cycle.

To derive (13-10), use (13-9) to establish the boundary condition occuring as the FET enters saturation ($\theta = 3\pi/2 - y_s$) and then use (13-3) to eliminate I_{DQ}:

$$i_{D,active}(3\pi/2 - y_s) = i_{D,sat}(3\pi/2 - y_s)$$

$$I_{DQ} - I_{DD} \sin(3\pi/2 - y_s) = \frac{[V_{DD} + V_{dm} \sin(3\pi/2 - y_s)]}{R_{on}}$$

$$(-\cos y + \cos y_s)\, I_{DD}\, R_{on} = V_{DD} - V_{dm} \cos y_s \ .$$

Deriving (13-11) is not as easy. First, determine the fundamental-frequency component I_{dm1} of $i_D(\theta)$, using a Fourier integral similar to that in (13-5):

$$I_{dm1} = \frac{1}{\pi}\int_0^{2\pi} i_D(\theta) \sin\theta\, d\,\theta$$

$$= \frac{2}{\pi}\int_{\frac{3\pi}{2} - y}^{\frac{3\pi}{2}} [I_{DQ} - I_{DD} \sin\theta] \sin\theta\, d\,\theta$$

$$- \frac{2}{\pi} \int_{\frac{3\pi}{2} - y_s}^{\frac{3\pi}{2}} [I_{DQ} - I_{DD} \sin \theta] \sin \theta \, d\theta$$

$$+ \frac{2}{\pi} \int_{\frac{3\pi}{2} - y_s}^{\frac{3\pi}{2}} \left[\frac{V_{DD} + V_{dm} \sin \theta}{R_{on}} \right] \sin \theta \, d\theta$$

$$= I'_{dm1} - I''_{dm1} + I'''_{dm1} \quad .$$

Now I'_{dm1} is what would be produced if saturation did not occur, and can be obtained by dividing (13-6) by $-R$:

$$I'_{dm1} = - \frac{I_{DD}}{2\pi} (2y - \sin 2y) \quad .$$

The second integral cancels the contribution of the unsaturated waveform during the saturation interval, and the third integral represents the contribution from the waveform during saturation. The second and third integrals can be combined prior to carrying out the actual integration; (13-3) is used to eliminate I_{DQ}:

$$I'''_{dm1} - I''_{dm1} = \frac{2}{\pi} \left(\frac{V_{DD}}{R_{on}} - I_{DQ} \right) \int \sin \theta \, d\theta + \frac{2}{\pi} \left(\frac{V_{dm}}{R_{on}} + I_{DD} \right) \int \sin^2 \theta \, d\theta$$

$$= \frac{2}{\pi} \left(\frac{V_{CC}}{R_{on}} + I_{DD} \cos y \right) \int \sin \theta \, d\theta + \frac{1}{\pi} \left(\frac{V_{dm}}{R_{on}} + I_{DD} \right) \int (1 - \cos 2\theta) \, d\theta$$

Now

$$\int \sin \theta \, d\theta = - \cos(3\pi/2) + \cos(3\pi/2 - y_s)$$

$$= - \sin y_s$$

$$\int (1 - \cos 2\theta) \, d\theta = (3\pi/2) - (3\pi/2 - y_s)$$

$$- \tfrac{1}{2} \sin(3\pi) + \tfrac{1}{2} \sin(3\pi - 2y_s)$$

$$= y_s + \tfrac{1}{2} \sin 2y_s \quad .$$

Combining the above,

$$I_{dm1} = \frac{1}{2\pi} \{- I_{DD}(2y - \sin 2y)$$

$$- 4 (V_{DD}/R_{on} + I_{DD} \cos y) \sin y_s$$

$$+ (V_{dm}/R_{on} + I_{DD})(2y_s + \sin 2y_s)\} \quad .$$

Since $V_{dm} = - I_{dm1} R$, multiply the above by $-R$ and collect all terms associated with V_{dm} on the left side:

$$V_{dm} \left[1 + \frac{2y_s + \sin 2y_s}{2\pi R_{on}}\right] = \frac{R}{2\pi} [I_{DD}(2y - \sin 2y)$$

$$+ 4 (V_{DD}/R_{on} + I_{DD} \cos y) \sin y_s$$

$$- I_{DD}(2y_s + \sin 2y_s)] \quad .$$

Division of both sides by the coefficient of V_{dm} produces (13-11).

13-2.2 Fortunately, the author used $R_{on}/R = 2.12/21.2 = 0.1$, and the problem can be solved by normalization of the values in Figure 13-3. (The reader curious about the three-place accuracy should realize that the author has the table from which Figure 13-3 was plotted.) In that figure, both supply voltage and load resistance are normalized to $V_{DD} = R = 1.0$ and the driven component of the drain current varies. In the present problem, the driven current is fixed and the supply voltage varies. It is therefore necessary for the normalized driven current to vary to allow for variation of the real V_{DD}. Hopefully this will become clear through the use of the following table.

Inspection of Figure 13-3 shows that maximum efficiency occurs with normalized driven current $I'_{DD} \cong 3.4$ A. Since $V_{DD} = 29$ is in the center of the range of V_{DD}, we place

R'	1.0	1.0	1.0	units
V_{DD}	28	29	30	V
V'_{DD}	1.0	1.0	1.0	V
I'_{DD}	3.52	3.4	3.29	A
I_{DD}	4.65	4.65	4.65	A
I_{DQ}	-1.19	-1.19	-1.19	A
I'_{DQ}	-0.9	-0.9	-0.9	A
V'_{om}	0.9		0.88	V
P_{max}	0.098		0.097	-
η	72		72	%
y	1.3		1.294	rad
	74.5		74.1	deg
y_s	0.5		0.440	rad
	28.6		25.2	deg
V_{om}	25.2		26.4	V
P_o	14.98		16.44	W
P_i	20.8		22.8	W
I_{dc}	0.743		0.761	A
$i_{D,max}$	2.84		2.94	A
$v_{D,max}$	53.8		57.7	V

I'_{DD} = 3.4 below V_{DD} = 29 in the normalization table. The values of I'_{DD} corresponding to V_{DD} = 28 and V_{DD} = 30 are obtained by recalling from Example 13-2.1 that $I_{DD} = I'_{DD}(V_{DD}/R)$, hence I'_{DD} must vary inversely with V_{DD} in the normalization. Appling the same relationship produces I_{DD} = 4.65 A, independent of V_{DD}, as expected. Since the driven and quiescent currents must be set to provide the required output at the lowest supply voltage, use I'_{DD} = 3.52 corresponding to V_{DD} = 28 to determine I_{DQ}:

$$I_{DQ} \quad \frac{I'_{DQ}}{I'_{DD}} I_{DD} = \left(\frac{-0.9}{3.52}\right)(4.65) = -1.19 \text{ A.}$$

The value of I'_{DQ} is fixed in the figure and the values of V'_{om}, η, y, y_s, and P_{max} for each V_{DD} are found by using the corresponding values of I'_{DD}.

The true output voltage is obtained by scaling; for example,

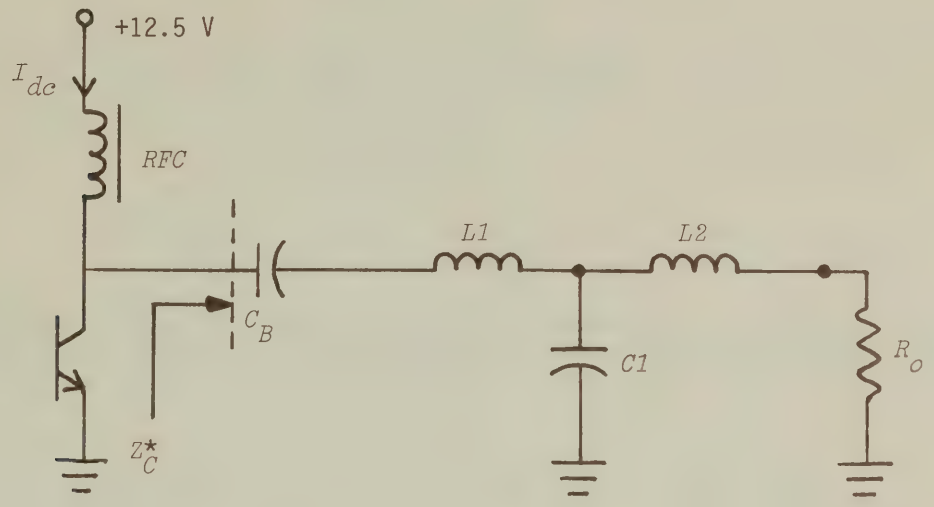
$$V_{om} = 28\ V'_{om} = 25.2 \text{ V,}$$

and the true output power is then $P_o = V_{om}^2/2R$. Input power is calculated using η, thus $P_i = P_o/\eta$, and $I_{dc} = P_i/V_{DD}$. The peak drain current is determined from I_{DD}, y, and y_s using (13-13). The peak drain voltage is then determined from P_{max}, P_o, and $i_{D,max}$:

$$v_{D,max} = \frac{P_o}{P_{max}\, i_{D,max}} \quad .$$

A preferable maximum voltage rating is, of course, 2 $V_{DD,max}$ = 60 V. If this were a real design probelm rather than a homework problem (or if R_{on}/R did not match Figure 13-3), the author would write a program and obtain the required numbers through iteration.

13-3.1 The circuit for this amplifier must provide a high impedance to harmonic components of the collector voltage as well as the required collector load impedance $Z_C^* = 2.5 + 0.2\ \Omega$. A T network can have such characteristics:



The reactance of RFC should be considerably greater than 2.5 Ω; for $X_{RFC} \geq 250\ \Omega$, $RFC \geq 0.23\ \mu H$. The reactance of C_B should be considerably less than 2.5 Ω; for $X_{CB} \leq 0.025\ \Omega$, $C_B \geq 440$ pF, hence use 0.001 μF and 10 pF in parallel.

The T network is designed to match R_o to 2.5 Ω (resistive); $L1$ is then increased slightly to provide the 0.2 Ω inductive reactance. Using [26] and assuming $Q = 5$ and $R_o = 50\ \Omega$,

$$A = R_C(1 + Q^2) = 65$$

$$B = \sqrt{(A/R_o) - 1} = 0.548$$

$$X'_{L1} = R_C\, Q = 12.5\ \Omega$$

$$X_{L2} = R_o\, B = 27.4\ \Omega$$

$$X_{C1} = A/(Q + B) = 11.72\ \Omega$$

We now note that

$$X_{L1} = X'_{L1} + 0.2 = 12.7\ \Omega\ .$$

Adequate rejection of harmonic currents is expected, since X_{L1}(at $2f$ = 350 MHz) $\geq$ 10 $|Z_C|$ (at f = 175 MHz). The component values are then

$$L1 = 11.6\ \text{nH}$$

$$L2 = 24.9\ \text{nH}$$

$$C1 = 77.6\ \text{pF}\ .$$

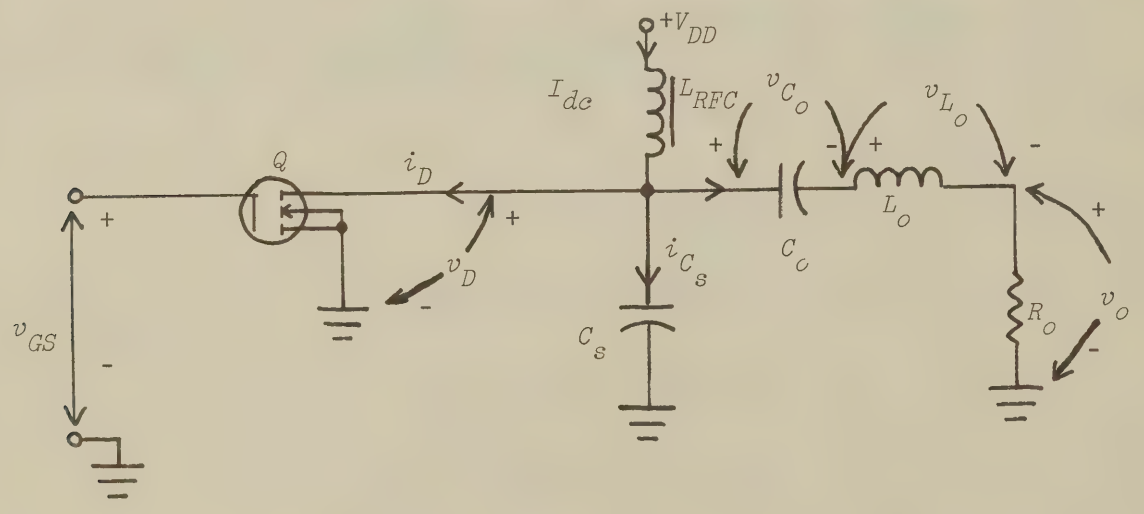
Finally, η = 0.6, V_{CC} = 12.5 V, and P_o = 40 W imply

$$P_i = 66.7\ \text{W}$$

$$I_{dc} = 5.33\ \text{A}\ .$$

The transistor will normally be within ratings if it is specified for P_o = 40 W with V_{CC} = 12.5 V. One may expect $v_{C,max} \cong 3\ V_{CC}$ = 37.5 V and $i_{C,max} \cong 3\ I_{dc}$ = 16 A. The student who thinks that "this is all there is" to designing a Class C mixed = mode PA will find out otherwise in the laboratory.

13-3.2 The circuit and relevant voltages and currents are as follows:



The model of the FET is

$$i_D = \begin{cases} 0, \text{ cutoff} \\ g_m(v_{GS} - V_T), \text{ active} \\ v_D/R_{on}, \text{ saturated} \end{cases}$$

(a) The computations sample operation of the amplifier at times

$$t = k\,\Delta t \ ,$$

where

$$\Delta t = 1/Kf \ ,$$

and K is an integer (typically $100 \leq K \leq 25$). It will be necessary to do computations for $k = 1, 2, \cdots, NK$, where $N \geq 10$ RF cycles. Inital conditions ($k = 0$):

$$v_{L_o}(0) = v_o(0) = 0$$

$$v_D(0) = v_{C_o}(0) = V_{DD}$$

$$i_D(0) = I_{dc}(0) = i_{C_s}(0) = i_o(0) = 0 \ .$$

$$k = 0$$

The program steps are as follows:

1. $k = k + 1$
2. $v_{GS}(k) = V_{GS} + V_{gs} \sin(2\,\pi\, f\, k\, \Delta t)$
3. $i_D(k) = g_m[v_{GS}(k) - V_T]$
4. If $i_D(k) > v_D(k\text{-}1)/R_{on}$, then $i_D(k) = v_D(k\text{-}1)/R_{on}$
5. If $i_D(k) < 0$, then $i_D(k) = 0$
6. $I_{dc}(k) = I_{dc}(k - 1) + (\Delta t/L_{RFC})[V_{DD} - v_D(k\text{-}1)]$
7. $i_o(k) = i_o(k - 1) + (\Delta t/L_o)\, v_{L_o}(k\text{-}1)$
8. $i_{C_s}(k) = I_{dc}(k) - i_o(k) - i_D(k)$

9. $v_o(k) = i_o(k)R_o$

10. $v_D(k) = v_D(k - 1) + (\Delta t/C_D)\, i_{C_s}(k - 1)$

11. $v_{C_o}(k) = v_{C_o}(k - 1) + (\Delta t/C_o)\, i_o(k - 1)$

12. $v_{L_o}(k) = v_D(k) - v_{C_o}(k) - v_o(k)$

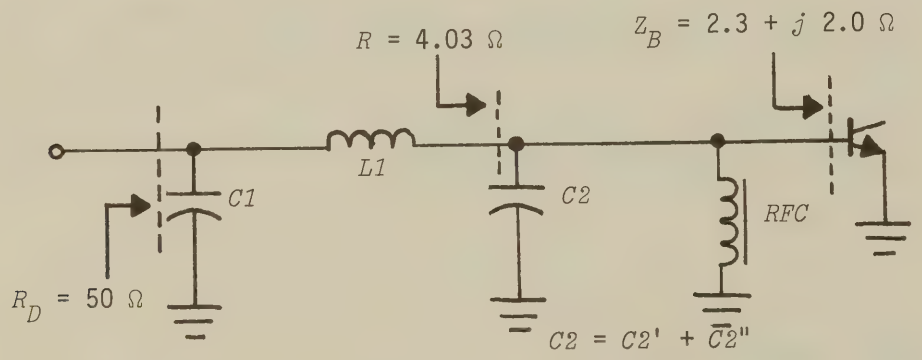
13. If $k < NK$, go to 1

14. Print out a waveform and/or perform integrals over the next RF cycle to determine I_{dc} (average), etc., print peak voltage and current values, etc.

15. STOP

(b) The actual programming will depend upon the facilities available. The instructor might assign different parameters to each student and use the results from all students to plot a graph of the performance of this amplifier as some parameter is varied.

13-4.1 A suitable circuit (from Figure 13-7d) is as follows:



The reactance of RFC should be considerably higher than $|Z_B|$, thus for $X_{RFC} \geq 100\ |Z_B| = 305\ \Omega$, $L_{RFC} \geq 0.28\ \mu H$.

First, note

$$Y_B = 1/Z_B = 0.248 - j\ 0.215 \text{ mho.}$$

Next set

$$B_{C2} = +\ 0.215 \text{ mho} \quad ,$$

hence

$C2$ = 196 pF.

This leaves a resistive impedance R = 1/0.248 = 4.03 Ω looking into the combination of $C2$ and Z_B.

The L network ($C1$ and $L1$) must match R_D = 50 Ω to R = 4.03 Ω. Adapting the parallel-to-series conversion formula in [26] yields

$$X_{C1} = \sqrt{\frac{R_D^2}{(R_D/R) - 1}} = 14.8\ \Omega$$

and

$$X_{L1} = R\,R_D/X_{C1} = 13.6\ \Omega \quad ,$$

hence $C1$ = 61.4 pF and $L1$ = 12.4 nH.

Finally, since G_p = 7.2 dB and P_o = 40 W, the driving power is

$$P_D = 40/10^{(7.2/10)} = 7.6\ \text{W}.$$

13-6.1 A square-law FET has the characteristic

$$i_D(\theta) = I_{DO}[1 - v_{GS}(\theta)/V_T]^2 \quad ,$$

and in Class A operation is biased so that

$$v_{GS}(\theta) = V_{GS} + V_{gs} \sin\theta \geq V_T$$

Observe that since $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$,

$$v_{GS}^2(\theta) = (V_{GS}^2 + \tfrac{1}{2} V_{gs}^2) + (2\,V_{GS}\,V_{gs}) \sin\theta$$

$$- (\tfrac{1}{2} V_{gs}^2) \cos 2\theta \ ,$$

$$i_D(\theta) = I_{D_o}\{(1 - 2\,V_{GS}/V_T + V_{GS}^2/V_T^2 + V_{gs}^2/2\,V_T^2\ )$$

$$+ (-2/V_T + 2\ V_{GS}/V_T^2)\ V_{gs} \sin\theta$$

$$+ (-V_{gs}^2/2\ V_T^2) \cos 2\theta\},$$

and the highest frequency component is at $2f$. (It is also apparent that the carrier-frequency term varies linearly with V_{gs}.)

Maximum power output and efficiency are obtained by adjusting the bias voltage so there is just enough quiescent current for Class A operation; therefore, set

$$V_{GS} = V_T + V_{gs} \quad .$$

Substituting this into the previous equation yields

$$i_D(\theta) = I_{DO}\left\{\left(\frac{3}{2}\frac{V_{gs}^2}{V_T^2}\right) + \left(\frac{2\ V_{gs}^2}{V_T^2}\right)\sin\theta - \left(\frac{V_{gs}^2}{2\ V_T^2}\right)\cos 2\theta\right\} \quad .$$

From the above, it is apparent that

$$I_{dc} = \frac{3}{2}\frac{V_{gs}^2}{V_T^2}\ I_{DO} \quad ,$$

$$I_{Dm1} = \frac{2\ V_{gs}^2}{V_T^2}\ I_{DO} = \frac{4}{3}\ I_{dc}$$

and

$$-I_{om} = I_{Dm2} = -\frac{1}{2}\frac{V_{gs}^2}{V_T^2}\ I_{DO} = -\frac{1}{3}\ I_{dc} \quad .$$

The load resistance R is chosen to produce a drain voltage swing $V_{Dm} = V_{DD}$, hence

$$P_o = \frac{V_{Dm}^2}{2\ R} = \frac{V_{DD}^2}{2\ R} = \frac{V_{Dm}(-I_{Dm2})}{2\ R} = \frac{V_{DD}\ I_{dc}}{6} \quad ,$$

and

$$\eta = \frac{P_o}{P_i} = \frac{1}{6} = 16.7\% \quad .$$

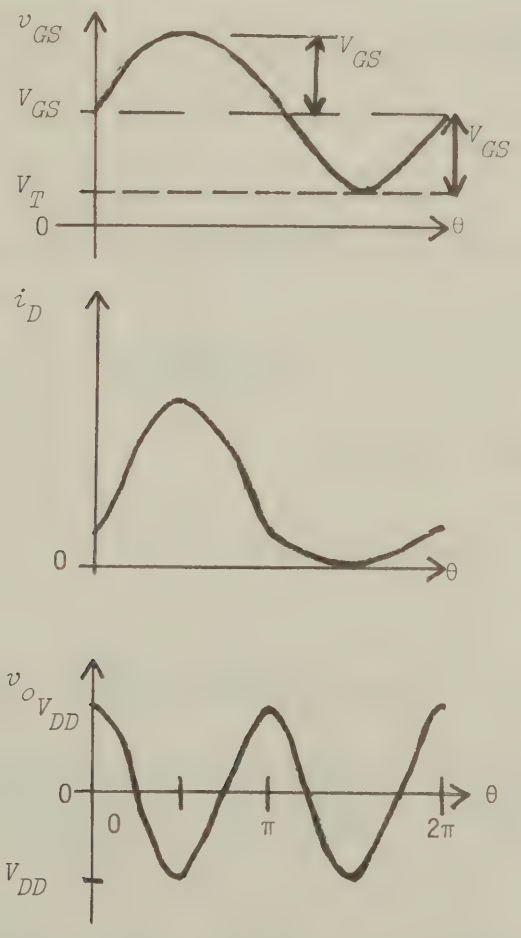
Other relevant parameters:

$$i_{D,max} = I_{dc} + I_{Dm1} - I_{Dm2} = \frac{8}{3} I_{dc}$$

$$v_{D,max} = 2\ V_{DD}$$

$$P_{max} = \frac{P_{o,max}}{v_{D,max}\ i_{D,max}} = \frac{V_{DD}\ I_{dc}/6}{(2\ V_{DD})(8/3\ I_{dc})} = 1/32 = 0.03125 \quad .$$

The circuit is that of Figure 13-1a. The waveform differ as shown:



13-6.2 First, use (13-3) to rewrite (13-1) in terms of $i_{C,max}$ and y:

$$i_{C,max} = I_{CQ} + I_{CD} = I_{CD}(1 - \cos y)$$

$$I_{CD} = \frac{1}{1 - \cos y} i_{C,max}$$

$$I_{CQ} = \frac{-\cos y}{1 - \cos y} i_{C,max}$$

The Fourier integral can be made somewhat simpler by moving the collector current peak to angular time $\theta = 0$, thus

$$i_C(\theta) = \begin{cases} \dfrac{i_{C,max}}{1 - \cos y}(-\cos y + \cos \theta), & 0 \leq \theta \leq y \text{ or } 2\pi - y \leq \theta \leq 2\pi \\ 0, \text{ otherwise} \end{cases} .$$

Symmetry requires that $i_C(\theta)$ contain only cosine components, thus if

$$A = \frac{2\, i_{C,max}}{\pi(1 - \cos y)} ,$$

the component at frequency $kf > f$ is

$$I_{cm,k} = A \int_0^y (-\cos y + \cos \theta) \cos k\theta \, d\theta$$

$$= A \left\{ -\cos y \int \cos k\theta \, d\theta + \frac{1}{2} \int \cos(k + 1)\theta \, d\theta + \frac{1}{2} \int \cos(k - 1)\theta \, d\theta \right\}$$

$$= A \left\{ -\frac{\cos y \sin ky}{k} + \frac{\sin(k + 1)y}{2(k + 1)} + \frac{\sin(k - 1)y}{2(k - 1)} \right\}$$

$$= A \left\{ -\frac{\cos y \sin ky}{k} + \frac{(k - 1)\sin(k + 1)y + (k + 1)\sin(k - 1)y}{2(k^2 - 1)} \right\}$$

$$= A \left\{ - \frac{\cos y \sin ky}{k} + \frac{k[\sin(k+1)y + \sin(k-1)y]}{2(k^2 - 1)} \right.$$

$$\left. - \frac{\sin(k+1)y + \sin(k-1)y}{2(k^2 - 1)} \right\}$$

$$= A \left\{ - \frac{\cos y \sin ky}{k} + \frac{k \sin ky \cos y}{k^2 - 1} \right.$$

$$\left. - \frac{\sin y \cos ky}{k^2 - 1} \right\}$$

$$= A \left\{ \frac{\cos y \sin ky - k \sin y \cos ky}{k(k^2 - 1)} \right\} \quad .$$

Substitution of A yields (13-17). Essentially the same procedure is used if the student elects to integrate the waveform given by (13-1).

13-6.3 A push-pull circuit similar to that of Figure 12-5 facilitates broadband operation by cancelling the even harmonics of the carrier frequency (in practice, the even harmonics are reduced by 10 to 20 dB from the levels present in an otherwise similar single-ended circuit). Since BJTs are to be used,

$$V_{eff} = V_{CC} - V_{sat} = 12 - 0.5 = 11.5 \text{ V}.$$

For $P_o = 5$ W,

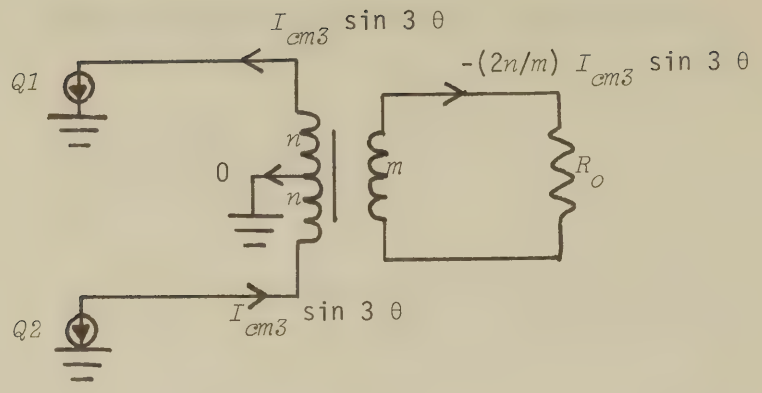
$$V_{om} = \sqrt{2R_o P_o} = \sqrt{2 \cdot 50 \cdot 5} = 22.4 \text{ V}$$

$$I_{om} = V_{om}/R_o = 0.447 \text{ A}.$$

Since $V_{cm} < V_{eff}$, a transformer turns ratio of $(m/n) = 2$ is selected and $V_{cm} = 11.18$ V.

To determine the collector currents, consider the equivalent circuit for the third harmonic:

P13-6.3



Harmonic currents (as opposed to transient waveforms) flow continuously from $Q2$ to $Q1$ and are therefore transformed from $2n$ primary turns to m secondary turns. Therefore

$$I_{cm,3} = (m/2n)\, I_{om} = 0.447 \text{ A.}$$

If using the transformer is confusing, the same result can be obtained by observing that each transistor must produce $\frac{1}{2} P_o = 2.5$ W, hence $2.5 = \frac{1}{2} V_{cm} I_{cm,3}$, hence $I_{cm,3} = 0.447$ A.

The ratio $I_{cm,3}/i_{C,max}$ is maximized with $y \cong 120/3 = 40°$. Using this in (13-17) yields

$$i_{C,max} = 2.423 \text{ A ,}$$

and (13-16) yields

$$I_{dc} = 0.712 \text{ A,}$$

hence

$$P_i = 12\, I_{dc} = 8.54 \text{ W}$$

and

$$\eta = 58.5\% \text{ .}$$

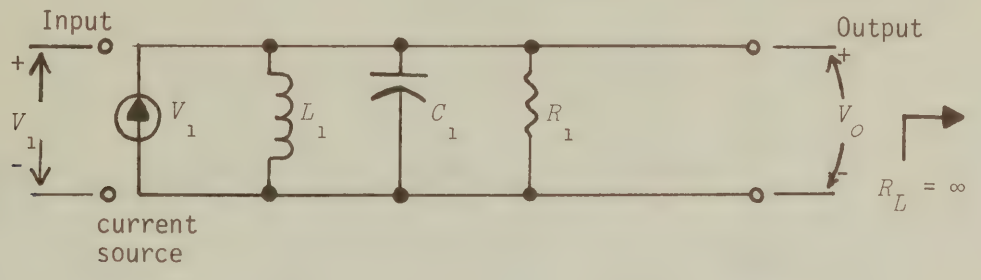
The peak voltage is

$$v_{C,max} = 12 + 11.18 = 23.18 \text{ V ,}$$

but a good design should allow for 24 V. While not part of the problem, it is interesting to note that if V_{CC} were reduced to 11.18 V and V_{sat} were zero,

$$P_{max} = \frac{5}{(22.36)(2.423)} = 0.0923 \quad .$$

13-7.1 While this rule of thumb is not strickly correct, it is widely used to make quick estimates of the Q of multi-section matching networks. Needless to say, the designer should finalize the estimates using one of the many computer-aided analysis programs that are now available. The nonrigorous derivation of this rule of thumb can be obtained by setting up a chain of tuned amplifiers that do not load each other; these have the form



To simplify things, let $R = 1$, $L_1 = 1/Q_1$, and $C_1 = Q_1$ so that resonance occurs at $\omega = 1$ and produces $V_o = V_1$. Simple network analysis then produces

$$T_1 = \frac{V_o}{V_1} = Y_1 = \frac{1}{R} + \frac{1}{j\omega L_1} + j\omega C_1$$

$$= 1 - \frac{j Q_1}{\omega} + j\omega Q_1$$

$$= 1 + j Q_1 (\omega - \frac{1}{\omega})$$

Now replace ω by $1 + \Delta\omega$:

$$T_1 \cong 1 + j Q_1 (1 + \Delta\omega - 1 + \Delta\omega) = 1 + j Q_1 (2 \Delta\omega)$$

$$|T_1|^2 \cong 1 - Q_1^2 (2 \Delta\omega)^2 \quad .$$

The squared amplitude transfer function of the network is then

$$|T_{net}|^2 \cong |T_n|^2 \, |T_{n-1}|^2 \cdots |T_2|^2 \, |T_1|^2$$

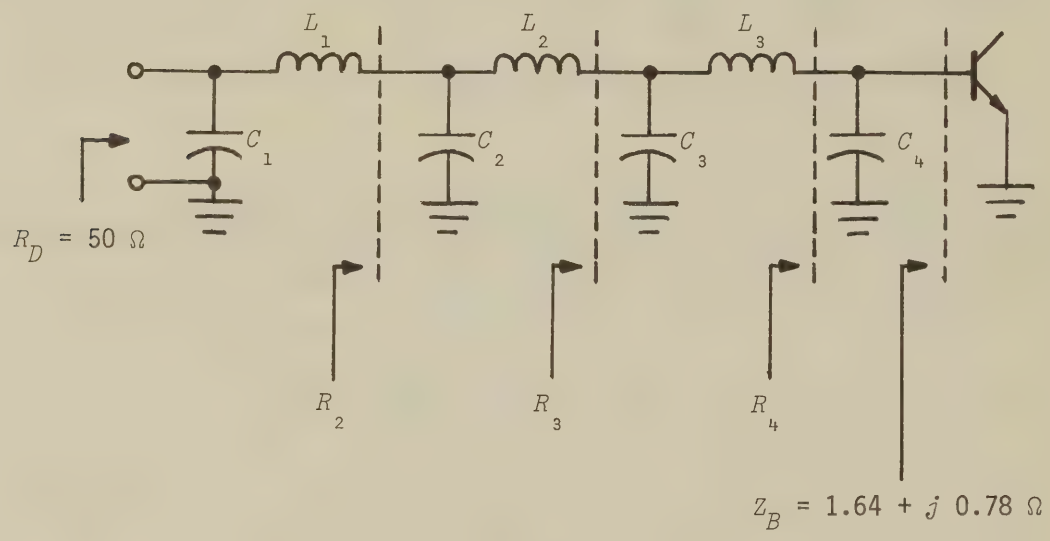
$$\cong 1 - Q_1^2(2\,\Delta\omega)^2 - Q_2^2(2\,\Delta\omega)^2 - \cdots - Q_n^2(2\,\Delta\omega)^2$$

$$\cong 1 - Q_{net}^2(2\,\Delta\omega)^2,$$

where

$$Q_{net}^2 = Q_1^2 + Q_2^2 + \cdots + Q_n^2 \quad .$$

13-7.2 The network has the following form:



The philosophy is to use C_4 to convert Z_B into a pure resistance R_4 and then to use a to-be-determined number of L-sections to convert R_4 to the desired driving impedance R_D = 50 Ω.

To determine C_4, first convert Z_B into Y_B, then choose C_4 to cancel the parallel conductance:

$$Q_B = X_B/R_B = 0.78/1.64 = 0.476$$

$$R_4 = 1/G_B = R_B(1 + Q_B^2) = 2.012\ \Omega$$

$$-B_{C_4} = B_B = X_B(1 + Q_B^2)/Q_B^2 = 0.237 \text{ mho}$$

For f = 175 MHz, C_4 = 216 pF.

The designer must now determine the number n of L-sections required to convert R_4 = 2.012 Ω into R_D = 50 Ω with $Q \leq 2.5$. Since minimum Q is achieved when all sections have the same Q_i, the intermediate impedances (R_2, R_3, etc.) form a geometric progression ($R_D/R_2 = R_2/R_3$, etc.). The following table is the generated with the aid of an immittance chart:

n	R_D	R_2	R_3	R_4	Q_i	Q
1	50	-	-	2.012	4.89	4.89
2	50	10.12	-	2.012	2.00	2.82
3	50	17.13	5.87	2.012	1.39	2.40

The Q_i of a section is the ratio of the reactance to the resistance at the intersection of the first intermediate resistance line and the $1/R_D$ = 20 mmho conductance line (this can be read directly if the immittance chart includes Q lines). The network Q is then $Q = Q_i\sqrt{n}$, using the result of the previous problem.

The particular problem requires three L-sections. The values of capacitive conductance and inductive reactance are read directly from the immittance chart and then converted to component values at the frequency of operation (175 MHz assumed):

B_{C_1} = 28 mmho $\qquad$ C_1 = 25.5 pF

B_{C_2} = 81 mmho $\qquad$ C_2 = 73.7 pF

B_{C_3} = 237 mmho $\qquad$ C_3 = 216 pF

X_{L_1} = 23.8 Ω $\qquad$ L_1 = 21.6 nH

X_{L_2} = 8.16 Ω $\qquad$ L_2 = 7.42 nH

X_{L_3} = 2.79 Ω $\qquad$ L_3 = 2.54 nH

13-7.3 The quarter wavelength line ($W1$) is used to provide the resistive component of the required impedance $Z_C^* = 3.3 + j\ 1.7$ Ω; the discrete element (inductor $L1$) is then used to provide the desired reactive component of the impedance Z_C^*:

TITLE Problem 13-7.2

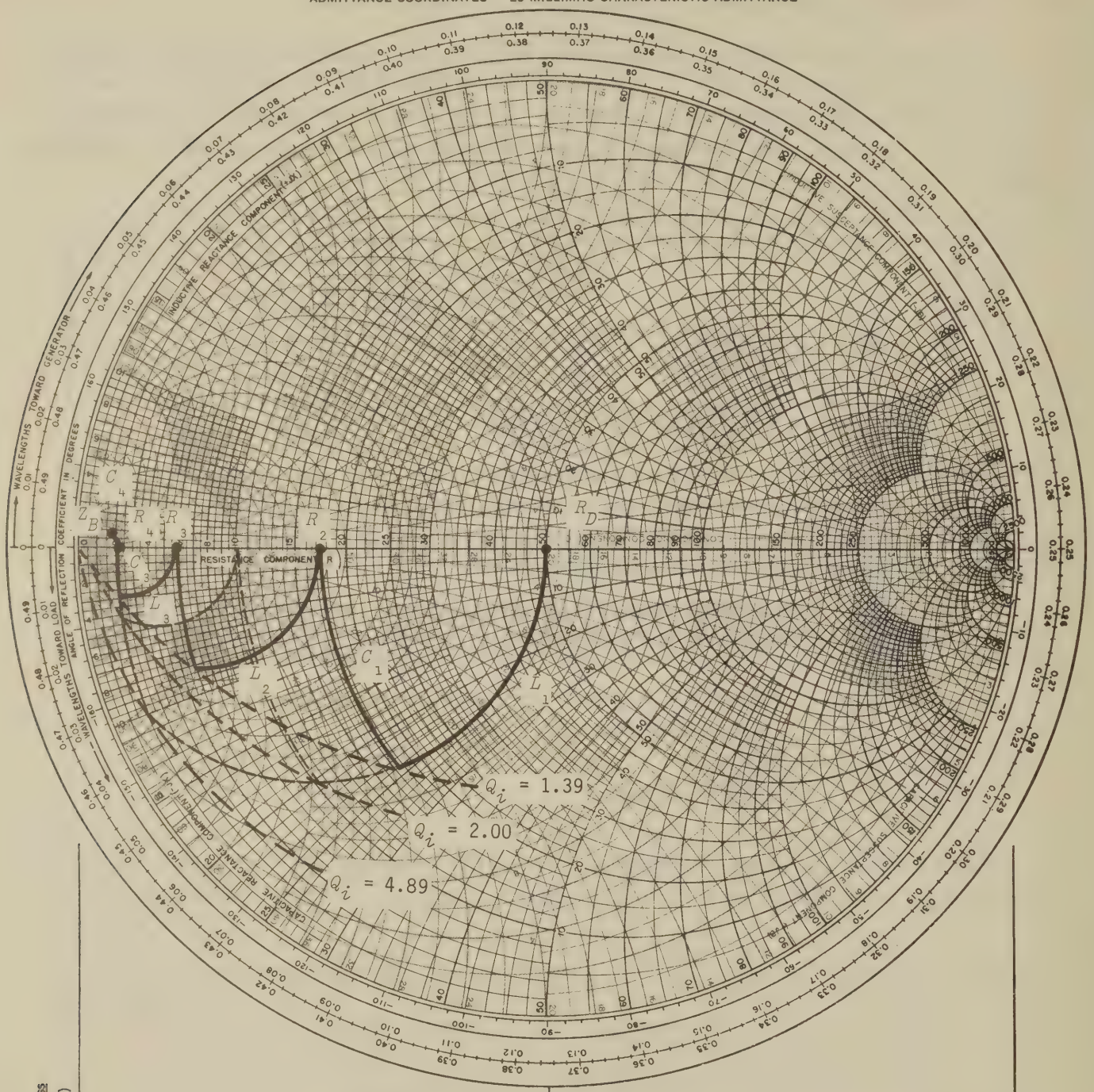
DATE

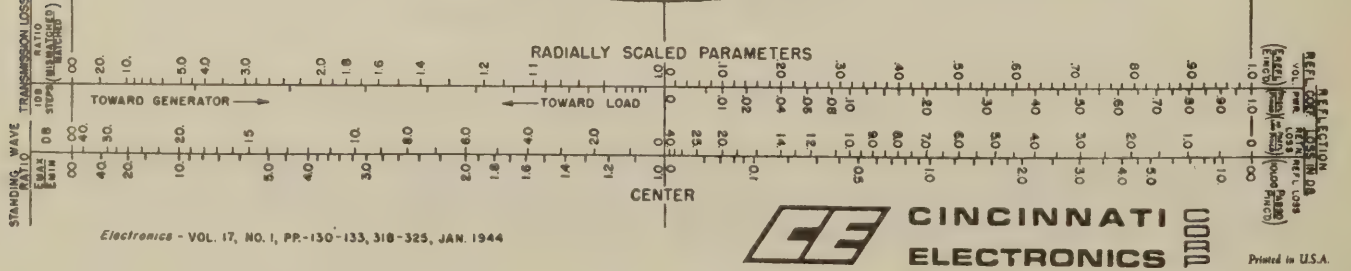
IMMITTANCE CHART

FORM 2192

IMPEDANCE COORDINATES — 50 OHM CHARACTERISTIC IMPEDANCE

ADMITTANCE COORDINATES — 20 MILLIMHO CHARACTERISTIC ADMITTANCE





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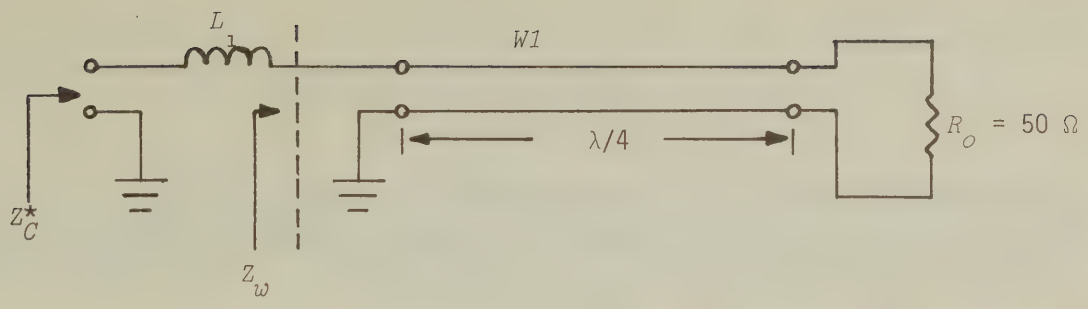


The characteristic impedance Z_o of line *W1* is chosen to convert R_o = 50 Ω to Z_w = 3.3 Ω, thus

$$Z_o = \sqrt{R_o Z_w} = \sqrt{50 \cdot 3.3} = 12.8\ \Omega\ .$$

Since X_{L_1} = 1.7 Ω at f = 175 MHz, L_1 = 1.55 nH.

Inspection of Figure 3-11 shows that for Z_o = 12.8 Ω and ε_r = 4.8, $W/H \cong 12$. Since H = 0.79 mm, $W \cong$ 9.5 mm, and Figure 13-12 gives $\varepsilon_{eff} \cong 4.3$. For free space, $\lambda_{FS} = 3 \cdot 10^8/175 \cdot 10^6$ = 171.4 cm, hence for the PC board,

$$\lambda_{PCB} = \lambda_{FS}/\sqrt{\varepsilon_{eff}} = 171.4/\sqrt{4.3} = 82.67\ \text{cm}.$$

The physical length of *W1* is then

$$l = \lambda_{PCB}/4 = 20.67\ \text{cm}.$$

The input impedance $Z_{w,k}$ of the line to a harmonic frequency kf is easily determined by noting that the electrical length of the line is either (a) A multiple of λ/2 and the line is therefore an impedance repeater, hence $Z_{w,k} = R_o$; or (b) an odd multiple of λ/4 and the line is therefore equivalent to a number of impedance repeaters and one quarter-wavelength transformer, hence $Z_{w,k}$ = 3.3 Ω. The impedance of L_1 at the frequency is calculated and added to Z_w, thus:

k	Z_w	X_{L_1}	$\lvert Z^*_{C,k} \rvert$
1	3.3	1.7	3.7
2	50	3.4	50.1
3	3.3	5.1	6.07
4	50	6.8	50.5
5	3.3	8.5	9.12

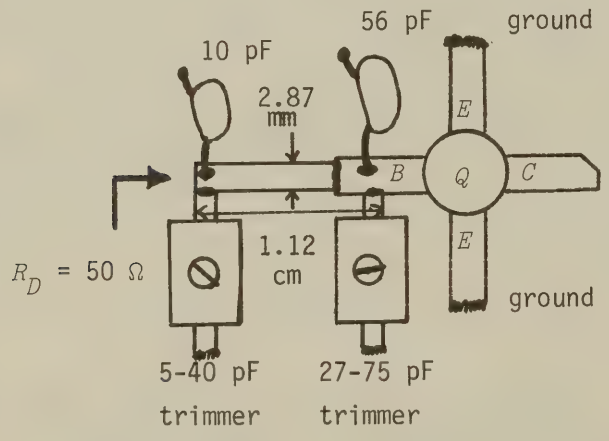
G_B = 497 mmho = 1/2.012 Ω.

The 237 mmho susceptance can be obtained at f = 465 MHz with a discrete capacitor C_1 = 81.1 pF, which is a practical value. To produce 237 mmho with a 50 Ω transmission line on this PC board would require $\gamma \ell$ = 0.236 λ, hence ℓ = 7.9 cm. Because this line would be rather long, the discrete capacitor is chosen.

Third, use *W1* with Z_o = 50 Ω to convert G_B = 497 mmho into Y_W = 20 mmho + j B_W. The immittance chart is used to determine that $\gamma \ell_w$ = 0.0335 λ, hence ℓ_w = 1.12 cm. Observe on the chart that B_W = -90 mmho.

Fourth, cancel the -90 mmho susceptance with a shunt capacitive susceptance. A discrete capacitor C_1 = 30.8 pF is chosen in preference to a transmission line, using the same reasoning as in the second step.

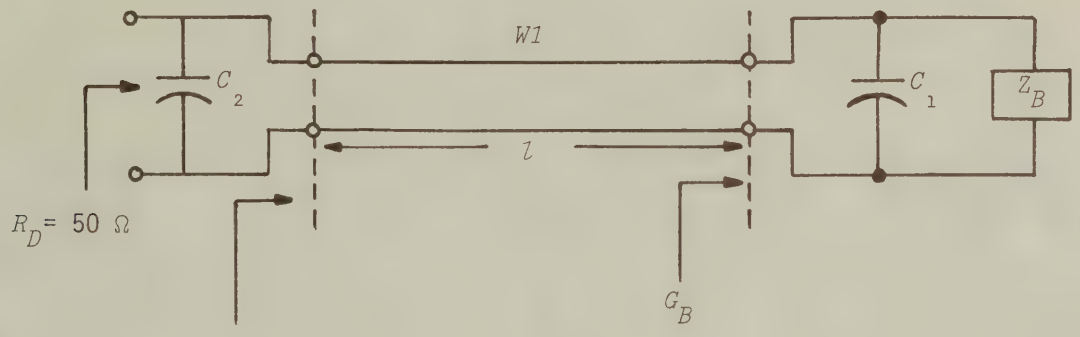
The resultant network might look like this:



(not to scale)

Many designers overlook the harmonic characteristics of transmission-line networks and then wonder why the performance differs from that with a discrete-element network. For example, the above network provides significantly less attenuation of the third and fifth harmonics than would a comparable discrete-element network.

13-7.4 As in this problem, the most practical design often incorporates both transmission lines and discrete elements. There are several possible approaches to this problem; the author's approach produced two discrete capacitors and a line with a convenient characteristic impedance:



$$Y_B = 20\ m\mho + j\ B_w$$

First, determine the characteristic impedance of a practical line. Example 13-7.2 suggests that $Z_o = 50\ \Omega$ is practical, since $W = 2.87$ mm. At 465 MHz, $\lambda_{FS} = 64.5$ cm and for W=2.87 mm, $\varepsilon_{eff} = 3.7$, hence

$$\lambda_{PCB} = 64.5/\sqrt{3.7} = 33.5 \text{ cm.}$$

The designer might also consider $Z_o = 10\ \Omega$. Using Figure 13-11, W/H=14, hence W= 2.23 cm. Using Figure 13-12, $\varepsilon_{eff} = 4.3$, hence $\lambda_{PCB} = 16.2$ cm. While the length may be nicer, the width of a line with $Z_o = 10\ \Omega$ appears excessive. A characteristic impedance of 50 Ω also conveniently coincides with printed immittance charts.

Second, determine Y_B from Z_B and use a shunt capacitive susceptance C_1 to cancel the susceptance component in Y_B, leaving a pure conductance G_B:

$$Y_B \cong 497 - j\ 237 \text{ mmho}$$

$$B_{C_1} \cong 237 \text{ mmho}$$

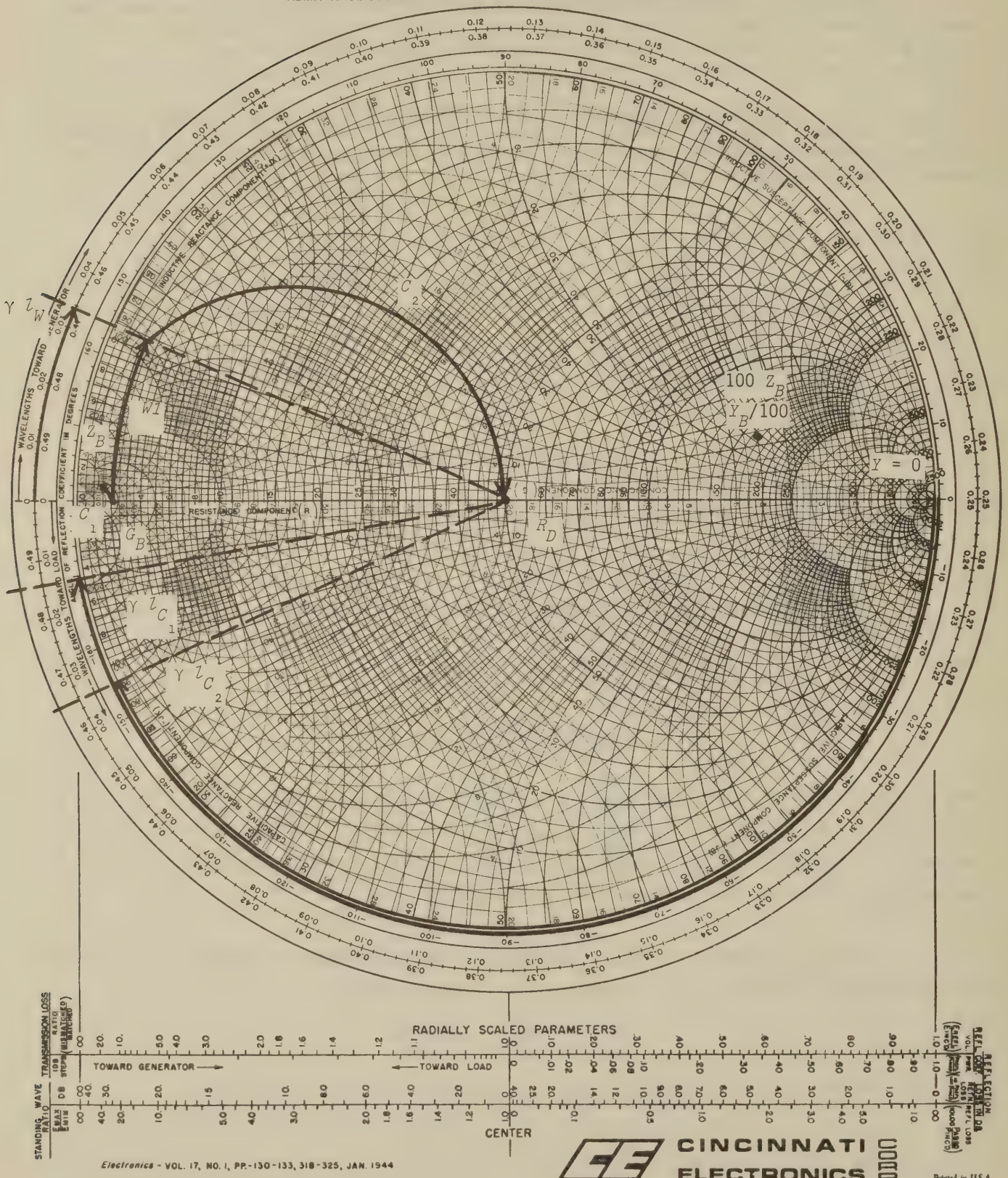
TITLE Problem 13-7.4 DATE

IMMITTANCE CHART

FORM 2192

IMPEDANCE COORDINATES — 50 OHM CHARACTERISTIC IMPEDANCE

ADMITTANCE COORDINATES — 20 MILLIMHO CHARACTERISTIC ADMITTANCE



RADIALLY SCALED PARAMETERS

TOWARD GENERATOR ⟶ ⟵ TOWARD LOAD

CENTER

STANDING WAVE RATIO

TRANSMISSION LOSS RATIO

REFLECTION LOSS

Electronics - VOL. 17, NO. 1, PP.-130-133, 318-325, JAN. 1944

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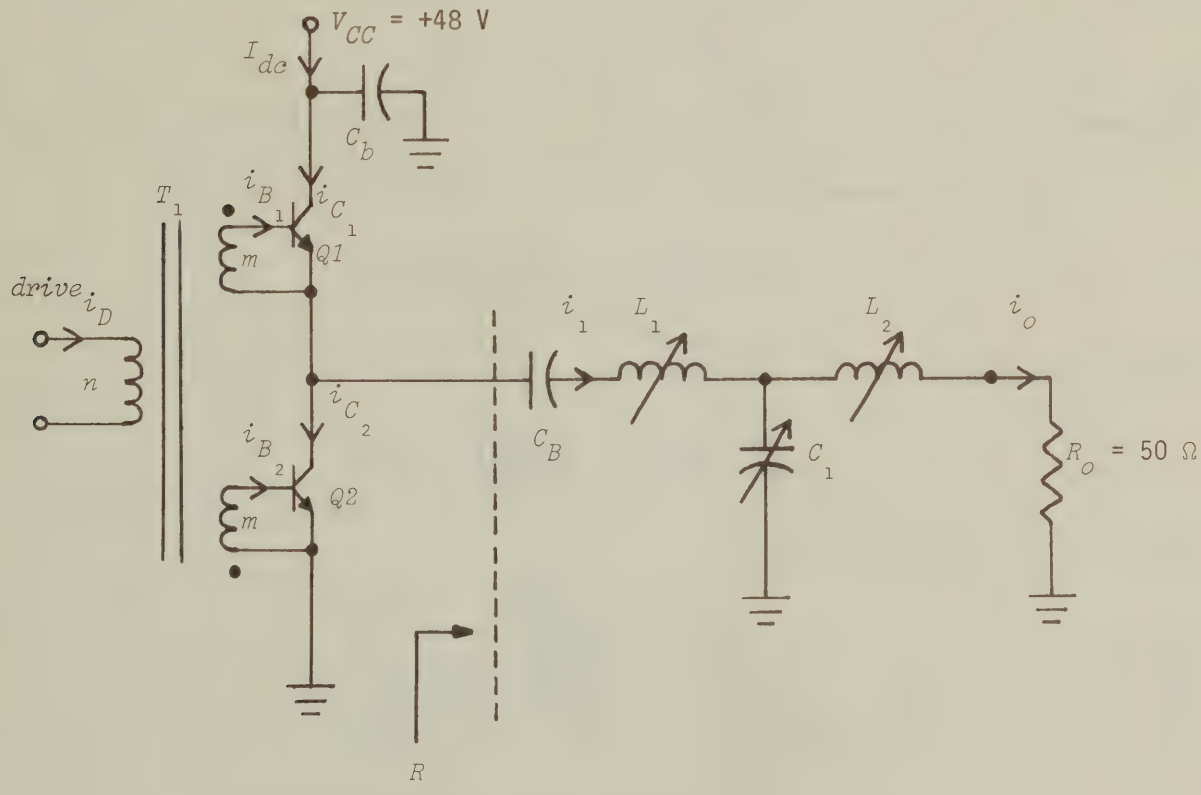
CHAPTER 14 PROBLEMS

P14-2.1

Since the student has presumably completed Chapters 12 and 13, the problems for Chapter 14 ask for more complete designs that include both the input and output circuitry. Since all Chapter 14 problems include drive and at least some "practical considerations," all are keyed to Section 14-2. The instructor can assign Problems 14-2.1, 14-2.2, and 14-2.5 as homework for Section 14-1 by directing the students not to design the input/drive portions of the PA. Similarly, Problems 14-2.3 and 14-2.4 can be used with Section 14-1 by omitting input/drive design and ignoring the effects of shunt collector capacitance. The instructor might then assign the unfinished aspects of the problems in connection with Section 14-2.

14-2.1 The circuit for this PA uses a T output network to provide both impedance transformation and high input impedance to harmonic frequencies:



First,

$$V_{eff} = V_{CC} - 2\,V_{sat} = 48 - 2 = 46 \text{ V},$$

hence using (14-5),

$$R = (2/\pi^2)(46^2)/100 = 4.29\ \Omega\ .$$

Using (14-4),

$$I_{cm} = I_{c,max} = \frac{2\ V_{eff}}{\pi\ R} = 6.83\ \text{A}$$

and using (14-6)

$$I_{dc} = (I_{cm}/\pi) = 2.17\ \text{A}\ ,$$

hence

$$P_i = 48 \cdot 2.17 = 104.3\ \text{W}$$

and

$$\eta = 95.9\%.$$

The peak voltage is $V_{c,max} = V_{CC} + V_{eff} = 94$ V, but a safe design would use $2\ V_{CC} = 96$ V.

The drive current is

$$i_{Bm} = I_{cm}/\beta = 6.83/20 = 342\ \text{mA},$$

and by using (14-16),

$$P_D = (2/\pi)(0.7)(0.342) = 152\ \text{mW}.$$

For $n = m = 1$, (14-15) yields

$$R_{DR} = \frac{(4/\pi)(0.7)}{0.342} = 2.61\ \Omega;$$

a more convenient value would be obtained with $n = 4$ and $m = 1$:

$$R_{DR} = \frac{(n/m)^2(4/\pi)(0.7)}{0.342} = 41.7\ \Omega.$$

Now for $X_{C_B} \leqq R/100$ at 1.8 MHz, $C_B \geqq 2.06\ \mu\text{F}$; therefore select a parallel combination of 2 μF, 0.1 μF, and 0.001 μF for C_B. A similar arrangement will suffice for C_b. The impedance of the T network should be at least 5 R at the second harmonic (3.6

MHz), hence

$$L_1 \geq (10 \cdot 4.29)/(2\,\pi \cdot 3.6 \cdot 10^6) = 1.9\ \mu\text{H}\ .$$

The T network can be designed as in Problem 13-3.1 or by using the immittance chart. Using the former method with $Q = 5$,

$$A = R(1 + Q^2) = 111.54$$

$$B = \sqrt{(A/R_o) - 1} = 1.11$$

$$X_{L_1} = Q\,R = 21.45\ \Omega$$

$$X_{L_2} = R_o\,B = 55.5\ \Omega$$

$$X_{C_1} = A/(Q + B) = 18.26\ \Omega$$

$$L_1 = 1.90\ \mu\text{H}$$

$$L_2 = 4.91\ \mu\text{H}$$

$$C_1 = 4840\ \text{pF}\quad .$$

14-2.2 To get $P_o = 5$ W in $R = R_o = 50\ \Omega$,

$$V_{dm} = V_{om} = \sqrt{2\,R_o\,P_o} = 22.4\ \text{V},$$

and

$$i_{D,max} = I_{dm} = I_{om} = V_{om}/R_o = 22.4/50 = 0.447\ \text{A}.$$

From (14-4),

$$V_{eff} = \frac{\pi\,R_o}{2}\,I_{om} = \frac{\pi}{2}\,V_{om} = 35.1\ \text{V}.$$

hence

P14-2.2

$$V_{DD} = \frac{R + R_{on}}{R} V_{eff} = \frac{50 + 2.5}{50} (35.1) = 36.8 \text{ V.}$$

and

$$v_{D,max} = 2 V_{DD} = 73.8 \text{ V.}$$

From (14-6),

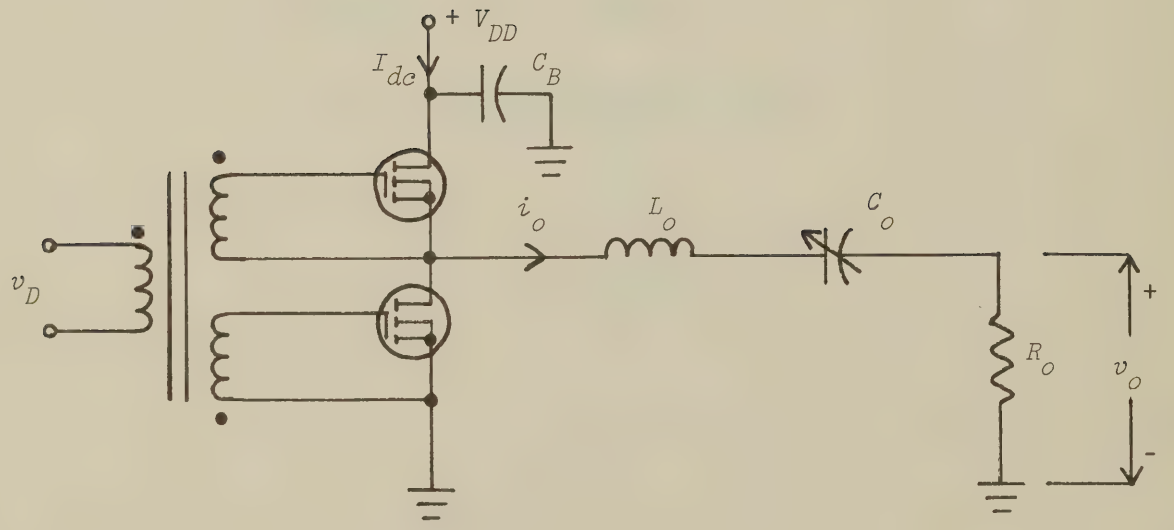
$$I_{dc} = \frac{2}{\pi^2} \frac{V_{eff}}{R} = 0.142 \text{ A}$$

hence

$$P_i = V_{DD} I_{dc} = 5.24 \text{ W}$$

and

$$\eta = 5/5.24 = 95.2\% .$$



To ensure saturation with I_{dm} = 0.5 A requires gate voltage of magnitude

$$V_{gsm} = I_{dm}/g_m = 0.5/0.1 = 5 \text{ V.}$$

Since this voltage is to be sinusoidal, the input transformer can be part of a tuned circuit.

For $Q = 5$, $X_{L_o} = -X_{C_o} = 250\ \Omega$; at $f = 27$ MHz, $L_o = 1.47\ \mu H$ and $C_o = 23.6$ pF. Since $R_{dc} = 36.8/0.142 = 257\ \Omega$, $X_{C_B} \leq 2.57\ \Omega$, hence $C_B \geq 0.0023\ \mu F$ and $C_B = 0.01\ \mu F + 100$ pF is chosen.

14-2.3 The requirement for operation over the entire 2 - 30 MHz range suggests the use of transmission-line transformers. The supply voltage V_{CC} will be reduced from 28 V to produce the desired output power. This will be accomplished by a high-efficiency class S modulator as discussed in Section 14-5 and Chapter 16.

First,

$$V_{eff} \leq 28 - 1 = 27\ ,$$

hence by use of (14-9)

$$R \leq \frac{8}{\pi^2}\frac{27^2}{150} = 3.94\ \Omega.$$

A 16:1 transformation can be obtained with wideband transformers, hence

$$R = 50/16 = 3.125\ \Omega\ ;$$

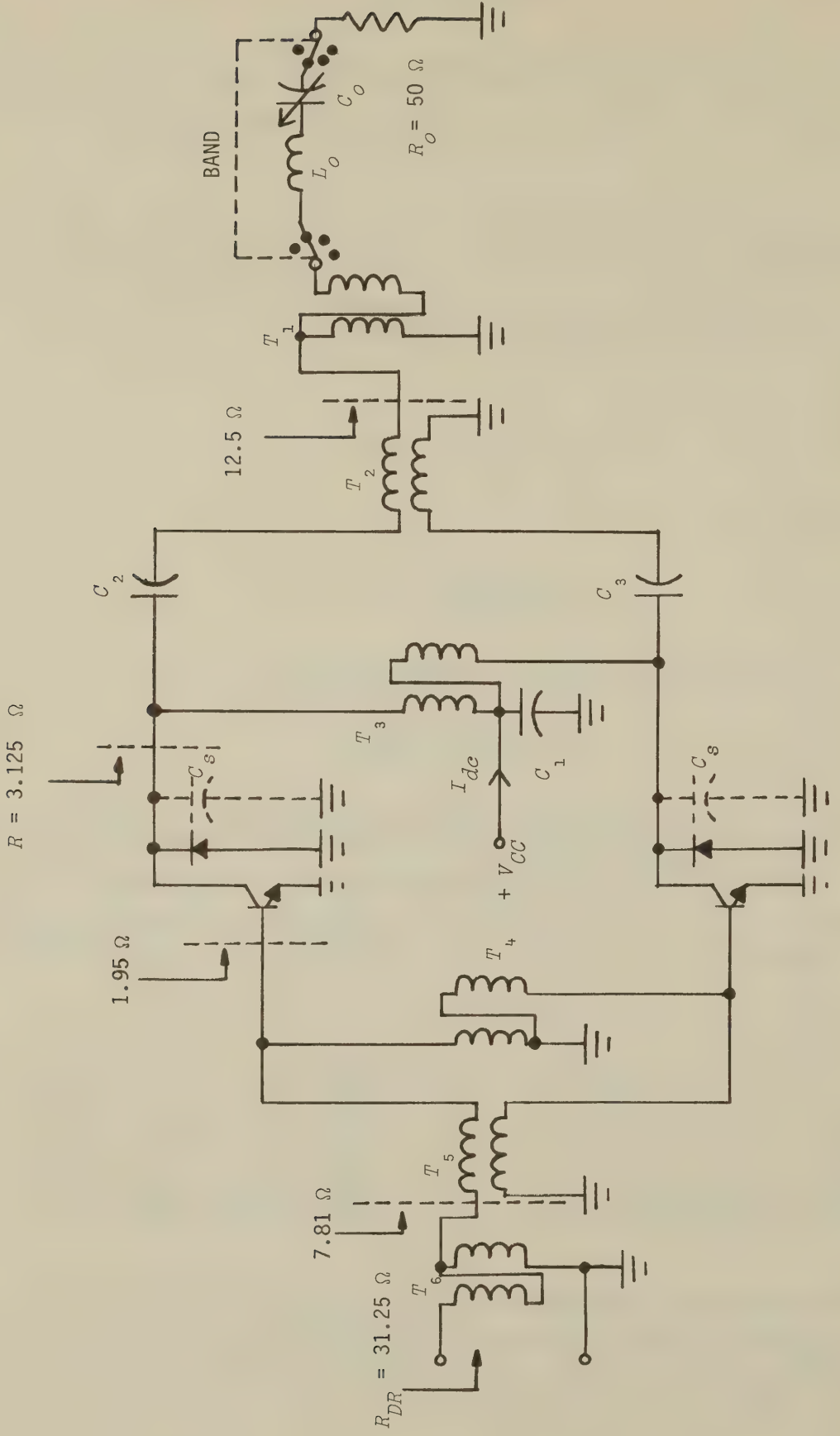
this requires three output transformers as shown on following page. The characteristic impedances of these transformers, obtained from Figure 12-13b and 12-12 are: T_1: 25 Ω; T_2 (balun): 12.5 Ω; T_3: 6.25 Ω. The 2 to 30 MHz range could be covered by a set of 4 tunable circuits (L_o - C_o) for the ranges 2-4, 4-8, 8-16, and 16-32 MHz. If the value of L_o is chosen to produce $Q = 5$ at the low end of each band, then $Q = 10$ at the high end of each band:

Band	X_{L_o}	L_o, μH	C_o, pF
2-4	250 Ω at 2 MHz	19.9	318 - 79.6
4-8	250 Ω at 4 MHz	9.95	159 - 19.9
8-16	250 Ω at 8 MHz	4.97	79.6 - 9.95
16-32	250 Ω at 16 MHz	2.49	39.8 - 4.97

The reactances of blocking capacitors C_1, C_2, and C_3 should be 3.125/100 Ω or less at 2 MHz, hence $C > 2.55\ \mu F$. Parallel combinatins of 10 μF, 0.1 μF, 0.001 μF, and 10 pF should work.

To get 150 W in $R = 3.125\ \Omega$ requires

P14-2.3


BAND
C_o
L_o
R_o = 50 Ω
T_1
12.5 Ω
T_2
C_2
C_3
T_3
C_1
I_{dc}
+ V_{CC}
C_s
R = 3.125 Ω
1.95 Ω
T_4
T_5
7.81 Ω
T_6
R_{DR} = 31.25 Ω

$$V_{eff} = \sqrt{\frac{\pi^2}{8}(3.125)(150)} = 24.0 \text{ V},$$

hence

$$V_{CC} = V_{eff} + V_{sat} = 25.0 \text{ V}.$$

The peak collector currents are

$$I_{cm} = \frac{4}{\pi}\frac{V_{eff}}{R} = 9.78 \text{ A},$$

and the dc current would be (from 14-10)

$$I_{dc0} = \frac{8}{\pi^2}\frac{V_{eff}}{R} = 6.23 \text{ A},$$

hence $P_{i0} = 25\ I_{dc0} = 155.6$ W if there were no capacitance shunting the collectors. The peak collector voltage is $v_{C,max} = 2\ V_{eff} = 50$ V.

The power dissipated because of the shunt capacitance is computed using (14-19), hence

f MHz	P_3 W	$P_i = P_s + P_{i0}$ W	$I_{dc} = P_i/V_{CC}$ A	η %
2	0.922	156.5	6.26	95.8
7.75	3.57	159.2	6.38	94.2
30	13.8	169.4	6.78	88.5

The heatsink should therefore be capable of dissipating at least $P_d = P_{i,max} - P_o = 19.4$ W.

Since $\beta = 15$, the peak base current is

$$I_{Bm} = I_{cm}/\beta = 9.78/15 = 0.652 \text{ A}.$$

By use of (14-15) and (14-16),

$$R_{DR\ (at\ base)} = \frac{4}{\pi}\frac{V_\gamma}{I_{Bm}} = 1.953\ \Omega \quad .$$

and

$$P_{DR} = \frac{2}{\pi} V_\gamma I_{Bm} = 0.415\ \text{W} \ .$$

A 25:1 impedance transformations would produce R_{DR} = 25(1.953) = 48.8 Ω; however, such a transformation (Figure 12-13c) would require 5 transformers. Since the driver can be designed to work into loads other than 50 Ω, it is convenient to use the three-transformer configuration shown to produce R_{DR} = 31.25 Ω. The characteristic impedance of the lines are: T_4: 3.91 Ω; T_5 (balun): 7.81 Ω; T_6: 15.62 Ω.

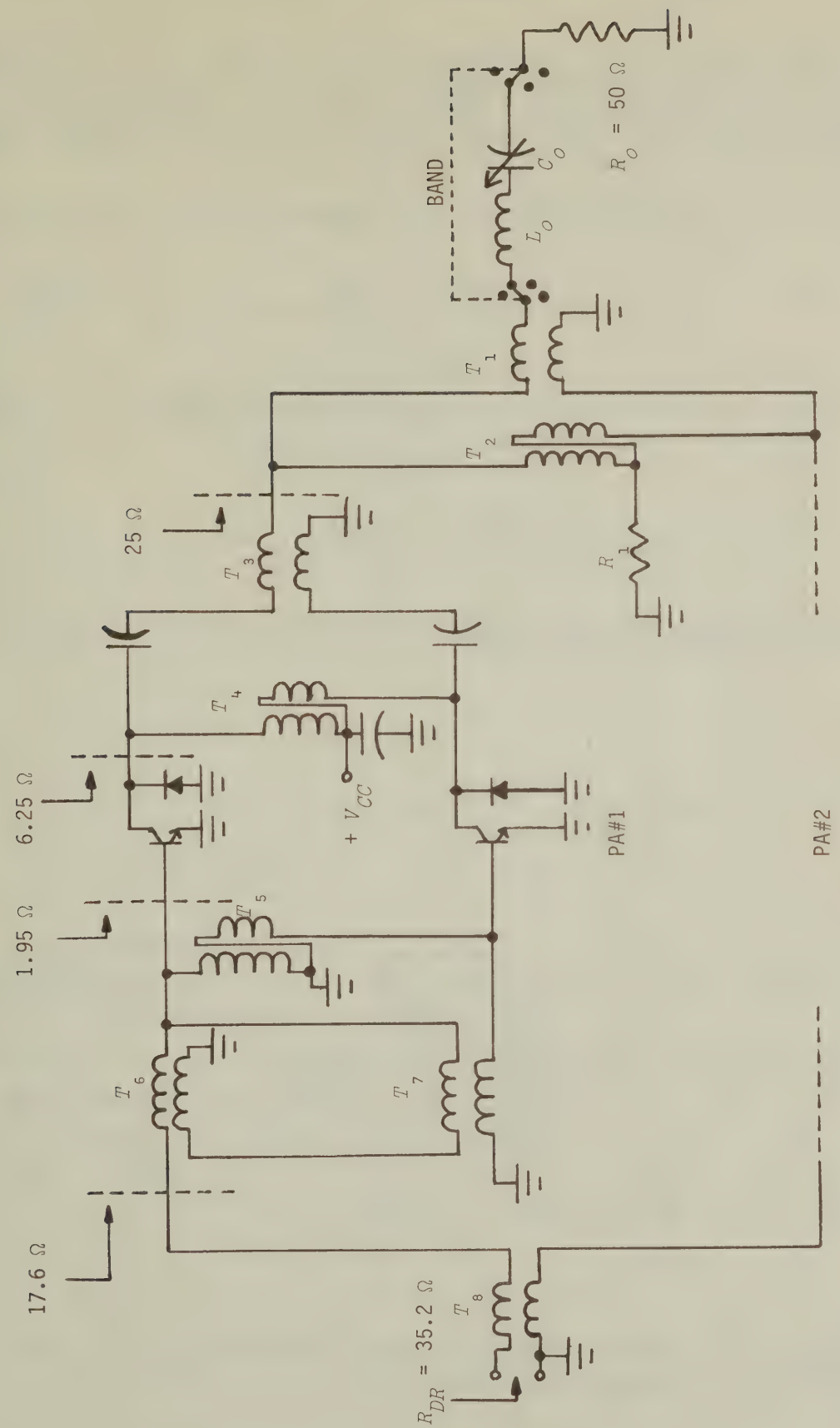
14-2.4 After completing the initial design of the PA for the high-efficiency marine-band SSB transmitter, the designer finds that either (a) Suitable transistors with a 10 A peak rating are not available or (b) Suitable transistors with a 5 A peak rating are less than half the price of those with a 10 A rating. All is not lost: two 75 W PAs can be combined to produce the required 150 W output. Since C_s for these transistors is exactly half that for those of Problem 14-2.4, most calculations need not be repeated.

Since Class D PAs define their voltage outputs over the entire RF cycle, and singe the voltage V_h appearing on T_2 is ideally zero, the power combining network can be included between the transistors and the output filter. This would not be possible if two single-ended PAs (Class C mixed-mode or Class E) were to be combined.

To get P_o = 75 W form a PA with V_{eff} = 24 V requires I_{cm} = 9.78/2 = 4.89 A and and R = (3.125)(2) = 6.25 ohms. This is quite convenient, since the broadband power combiner of Figure 12-15c has an input impedance of 25 ohms for each PA when the combined load is R_o = 50 ohms. From Figure 12-13b, the characteristic impedance of the lines in T_4 and T_3 are 12.5 ohms and 25 ohms, respectively. Balun T_1 must have a line with Z_o = 50 ohms. To prevent interaction of PA#1 and PA#2, R_1 = 12.5 Ω. As in Problem 12-7.3, the line in T_2 can have Z_o = 25 ohms, but this is noncritical.

On the input side, we discover that the 2:1 transformation produced by the power splitter can be combined with a 9:1 transformation from the configuration of Figure 12-13c to get an 18:1 transformation and R_{DR} = 35.2 ohms, almost what we had in the previous problem. If we set $R_h = \infty$ to ensure equal division of driving current between PA#1 and PA#2, we find that there is no current in the hybrid transformer, hence only a balun in needed. The characteristic impedances of the lines in the input transformers are then: T_5: 5.86 Ω; T_6: 5.86 Ω; T_7: 11.7 Ω; T_8: 35.2 Ω.

14-2.5 This problem illustrates both an application where high efficiency is desirable


BAND
C_O
L_O
R_O = 50 Ω
T_1
T_2
T_3
T_4
T_5
T_6
T_7
T_8
R_1
25 Ω
6.25 Ω
1.95 Ω
17.6 Ω
+ V_{CC}
PA#1
PA#2
R_{DR} = 35.2 Ω

and where a current-switching Class D PA provides the right output. The -500 ohm reactance is cancelled by a +500 ohm reactance from a "loading coil." Because the Q of that inductor is 100, it has a resistance of 5 ohms and produces a net load to the PA of R = 25 ohms. (It is not uncommon to low-frequency transmitters with short antennas to consume most of the transmitter power in the loading coil.)

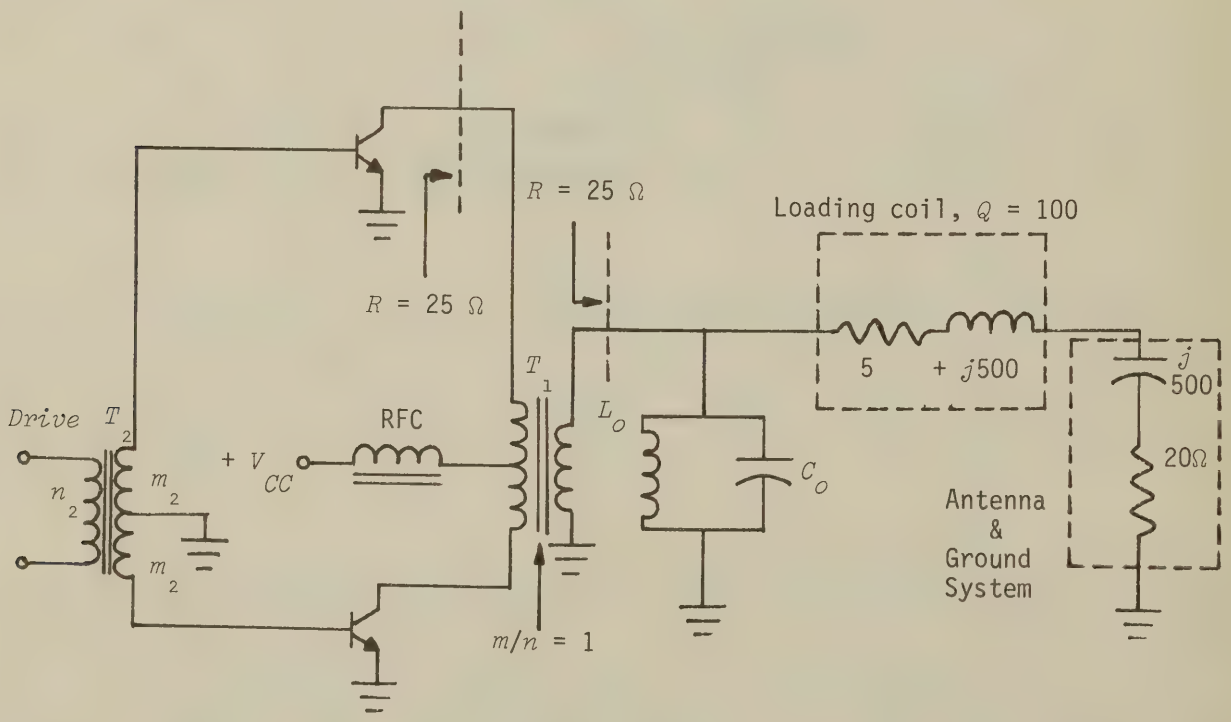
We begin by checking the capability of a complementary Class D PA for this application:

$$P_{o,max} = \frac{2}{\pi^2} \frac{V^2_{eff,max}}{R} = \frac{2}{\pi^2} \frac{23^2}{25} = 4.3 \text{ W} .$$

A transformer-coupled voltage-switching configuration similarly produces P_o = 17.2 W. However, for a current-switching configuration, (14-13) yields

$$P_{o,max} = \frac{\pi^2}{8} \frac{V^2_{eff,max}}{R} = \frac{\pi^2}{8} \frac{23^2}{25} = 26.1 \text{ W} ,$$

hence that configuration is selected. Note that this is not a "broadband" application and conventional transformers are sufficient.



to get P_o = 25 W requires lowering the supply voltage by means of a swithcing or series-pass regulator; rearranging (14-13) produces

$$V_{eff} = \sqrt{\frac{8}{\pi^2} \cdot 25 \cdot 25} = 22.5 \text{ V}$$

hence

$$V_{CC} = V_{eff} + V_{sat} = 23.5 \text{ V.}$$

From (14-12),

$$i_{C,max} = I_{dc} = \frac{\pi^2}{8} \frac{V_{eff}}{R} = \frac{\pi^2}{8} \frac{22.5}{R} = 1.11 \text{ A}$$

$$P_i = V_{CC} I_{dc} = 23.5 \cdot 1.11 = 26.1 \text{ W}$$

$$\eta = 25/26.1 = 95.7\% \text{ (for PA alone)}$$

$$\eta = (25/26.1)(20/25) = 76.6\% \text{ (including loading coil)}$$

$$v_{C,max} \cong \pi V_{CC} = 73.9 \text{ V.}$$

Now $X_{RFC} >> R$, hence for X_{RFC} = 2500 ohms at f = 200 khz,

$$L_{RFC} = 2 \text{ mH.}$$

Harmonic currents must not (and cannot) enter the antenna, hence a parallel-tuned circuit (L_o-C_o) is needed. For $Q \geq 5$, X_{L_o} = 25/5 = 5 Ω at f = 400 kHz, hence L_o = 1.99 μH. To tune the 200 to 400 kHz range, 0.0796 μF ≦ C_o ≦ 0.318 μF. Needless to say, C_o will be implemented with switch-selectable stepped values rather than just a "bread slicer."

The base current must be a squarewave with peak value

$$I_{Bm} = I_{dc}/\beta = 1.11/20 = 55 \text{ mA.}$$

Since V_γ = 0.7 V,

$$P_{DR} = V_\gamma I_{Bm} = 39 \text{ mW.}$$

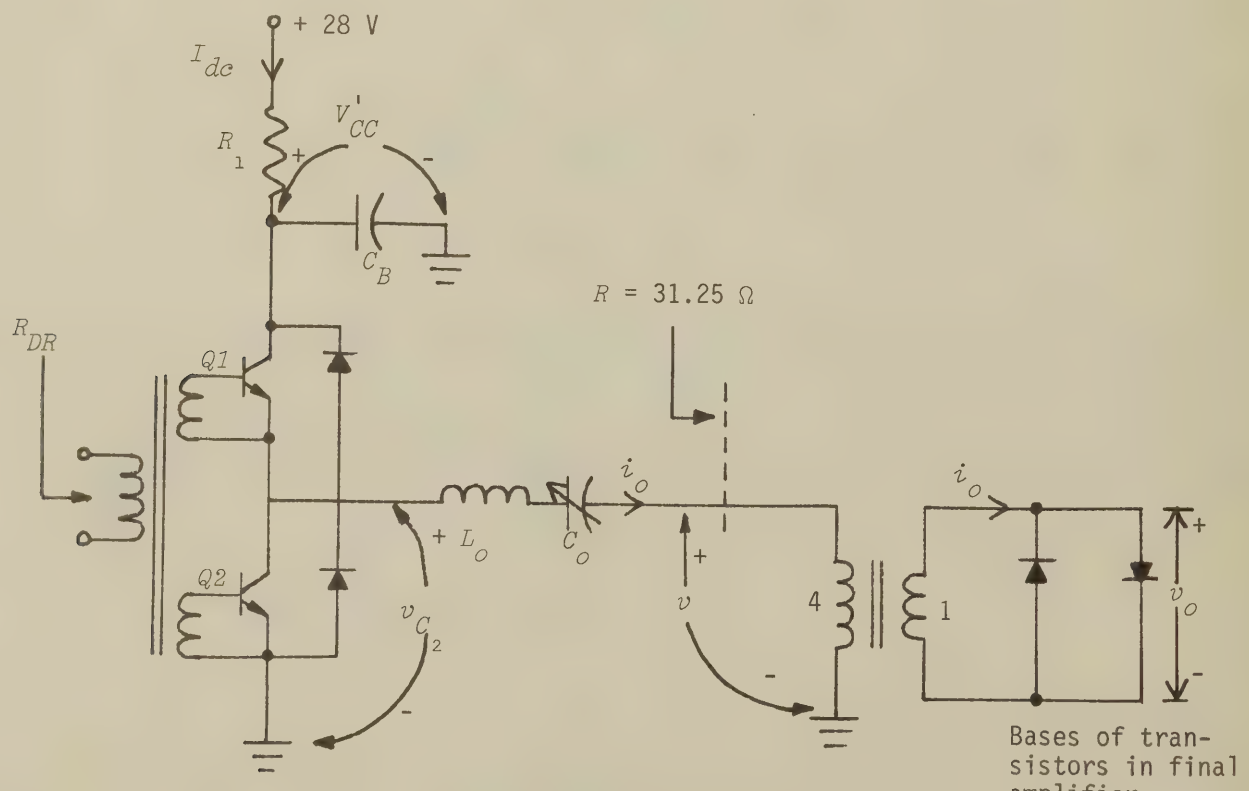
Since both the driving voltage and driving currents are squarewaves,

$$R_{DR} = \left(\frac{n_2}{m_2}\right)^2 \frac{V_\gamma}{I_{Bm}} = \left(\frac{n_2}{m_2}\right)^2 \cdot 12.7\ \Omega\ .$$

With $n_2/m_2 = 2$, $R_{DR} = 50.9\ \Omega$.

14-2.6 Designing a driver is not quite the same as designing a final power amplifier (see Problem 12-2.3 for one example). The astute student will recognize the load of this driver as the high-efficiency marine SSB PA designed in Problem 14-2.3. The single-frequency output circuit here serves only to simplify the problem; a bank of tuned circuits could be used to cover 2 to 30 MHz. However, a driver circuit more amenable to broadband operation will be designed in the next problem.

From Problem 14-2.3, we have (for the driver) P_o = 415 mW. With a 4:1 transformer in the input circuit to the final amplifier, R = (16)(1.953) = 31.25 ohms and I_m = $I_{om}/4$ = 652/4 = 163 mA for the driver. Similarly, the driver's load voltage v is a squarewave with peak amplitude $V_m = 4\ V_{om}$ = 4 V.



Since the fundamental-frequency component of v_{C_2} must be equal to the fundamental-frequency component of v, and since both are squarewaves, their peak-to-peak (ac) amplitudes must be equal and

$$V'_{eff} = 2\ V_m = 8\ \text{V}$$

hence $$V'_{CC} = V'_{eff} + 2\ V_{sat} = 8 + 2(0.5) = 9\ \text{V}.$$

Since $I_{cm} = I_m = 163$ mA,

$$I_{dc} = 51.9\ \text{mA}.$$

One must <u>not</u> attempt to use a voltage regulator to produce $V'_{CC} = 9$ V, since that voltage is a function of V_{sat} of $Q1$ and $Q2$ and V_γ of the load. One could use a current regulator to provide $I_{dc} = 51.9$ mA. However, since V'_{CC} is considerable smaller than the 28 V supply and since the drive power is a small fraction of the power consumed by the final power amplifier, it is convenient to use a resistor to accomplish the voltage drop. To allow $I_{dc} \geq 51.9$ mA, $R \leq (28 - 9)/0.0519 = 368$ ohms, hence a standard value $R = 330$ ohms is selected. This produces

$$I_{dc} = (28 - 9)/R = 19/330 = 57.6\ \text{mA}$$

$$P_i = 28(0.0576) = 1.61\ \text{W}$$

$$P_R = (19)(0.0576) = 1.094\ \text{W}\ \text{(hence } R \text{ should have a 2 W rating)}$$

$$I_{cm} = \pi\ I_{dc} = 181\ \text{mA} < 200\ \text{mA (rating)}$$

$$I_{om} = 4\ I_m = 723\ \text{mA (hence saturation is ensured)}$$

$$P_o = \left(\frac{4}{\pi}\right)(4)(0.181) = 0.460\ \text{W}$$

$$P_d = P_i - P_o - P_R = 55.4\ \text{mW (total, } Q1 \text{ and } Q2\text{)}$$

$$\eta_{driver} = P_o/P_i = 28.6\ \%$$

It is interesting to note that at midband (see Problem 14-2.3) that

$$\eta_{PA\ +\ driver} = 150/(159.2 + 1.61) = 93.3\ \%,$$

compared to 94.2% without the driver.

Since the load is another amplifier and not an antenna, $Q \cong 3$ is acceptable, hence at f = 10 MHz, $X_{L_o} = -X_{C_o} = 3(31.25) = 93.7$ ohms, thus L_o = 1.49 μH and C_o = 170 pF. To ensure an essentially constant V'_{CC}, $X_{C_B} \leq R_1/100 = 3.3$ ohms, hence $C_B \geq 0.005$ μF.

The transistors should have ratings of $v_{C,max}$ = 28 V or more, since that voltage can be applied to the devices if drive is removed. With β = 20,

$$I_{Bm} = I_{cm}/20 = 181/20 = 9.05 \text{ mA},$$

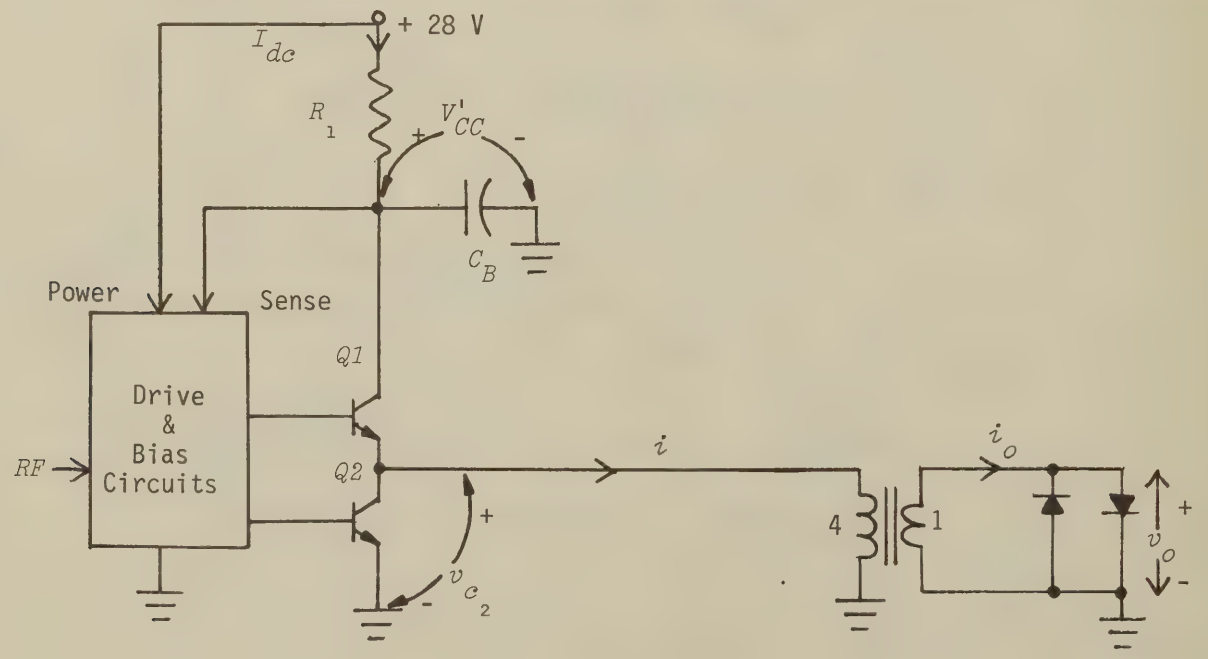
hence from (14-15) and (14-16),

$$P_{DR} = (2/\pi)(0.7)(0.009) = 4.0 \text{ mW}$$

$$R_{DR} = (n^2/m^2)(4/\pi)(0.7/0.009) = (n^2/m^2)\ 98.4\ \Omega\ .$$

If n/m = 2/3, R_{DR} = 43.7 Ω; however, 98 ohms is probably more convenient for this power level.

14-2.7 The previous driver design (Problem 14-2.6) made little use of the efficiency of Class D operation. One may wonder why one should bother with Class D operation of a driver. In many cases, use a Class B (or A) driver results in little or no decrease in transmitter efficiency and some simplification of circuitry, since output (driver-to-final amplifier) tuning is not required.



The class B driver could use either a complementary configuration (shown) or a push-pull configuration. The output and dc current requirements are unchanged. If the driving circuitry can produce the desired 181 mA output, then I_{dc} = 51.9 mA, I_{cm} = 163 mA, and P_i = 28(0.0159) = 1.45 W, hence this circuit could be slightly more efficient than the one of Problem 14-2.6. Since the output voltage v_o is a squarewave, the collector voltage v_{c_2} is also a squarewave, as in Problem 12-2.3.

To ensure linear operation of *Q1* and *Q2*, $V'_{CC} \geq 9$ V. Since I_{dc} = 51.9 mA is determined by the driving circuitry, R = 330 ohms produces

$$V'_{CC} = 28 - 330(0.0519) = 10.87 \text{ V}$$

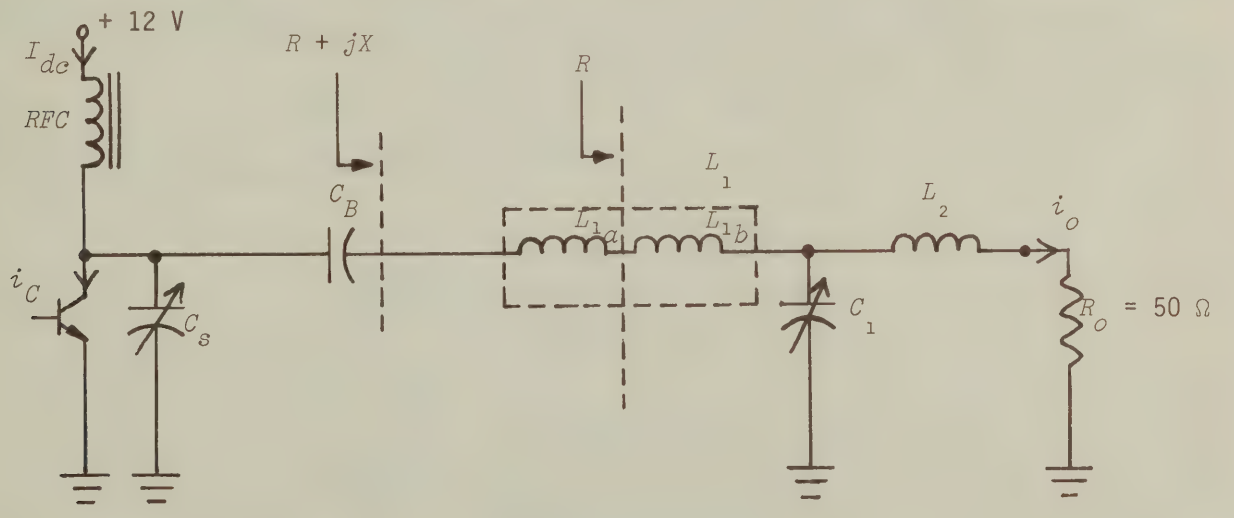
$$P_R = 0.889 \text{ W}$$

$$P_o = (2/\pi)(4)(0.163) = 0.415 \text{ W}$$

$$P_d = P_i - P_o - P_R = 149 \text{ mW}.$$

Regulation of the output current can be accomplished by monitoring I_{dc} or V'_{CC}.

14-3.1 A T-type matching network can be used to provide the collector load required for optimum Class E operation with the desired power output. Inductors L_{1a} and L_{1b} are physically implemented as a single inductor. Capacitor C_B is only for dc blocking.



First,

$$V_{eff} = 12 - 1 = 11 \text{ V}.$$

Now use (14-25) to determine

$$R \cong 0.577(V_{eff}^2/P_o) = 0.577(11^2/25) = 2.79\ \Omega$$

The high-Q assumption then produces

$$B = 0.1836/R = 0.0658\ \Omega,$$

hence

$$C_s = B/\omega = 201\ \text{pF (mid range of trimmer)}$$

and

$$X_{L_{1a}} = X = 1.152\ R = 3.22\ \Omega\ .$$

(The empirical adjustments for finite Q given in (14-26) and (14-27) apply only to a simple series-tuned circuit and may or may not be accurate for a T network.)

For $X_{RFC} \geqq 100\ R = 279\ \Omega$, $L_{RFC} \geqq 0.854\ \mu H$, hence $L_{RFC} = 1\ \mu H$ is selected. For $X_{C_B} \leqq R/100$, $C_B \geqq 0.11\ \mu F$. Since the factor of 100 is arbitrary, $C_B = 0.1\ \mu F + 0.001\ \mu F + 10\ pF$ is selected.

The T network consisting of C_1, L_2, and L_1 must transform 50 Ω into 2.79 Ω. Using the approach outlined in [26] of Chapter 13 with $Q = 5$.

$$A = R(1 + Q^2) = 2.79\ (1 + 5^2) = 72.5$$

$$B = \sqrt{(A/R_o) - 1} = 0.671$$

$$X_{L_{1a}} = R\ Q = 13.95\ \Omega$$

$$X_{L_2} = R_o\ B = 33.6\ \Omega$$

$$X_{C_1} = A/(Q + B) = 12.8\ \Omega$$

$$X_{L_1} = X_{L_{1a}} + X_{L_{1b}} = 17.17\ \Omega$$

L_1 = 52.6 nH

L_2 = 103 nH

C_1 = 239 pF (mid-range of trimmer)

For optimum Class E operation,

$$v_{C,max} = 3.56\ V_{CC} = 42.7\ \text{V}$$

$$I_{dc} = V_{eff}/1.734\ R = 2.27\ \text{A}$$

$$i_{C,max} = 2.86\ I_{dc} = 6.50\ \text{A}$$

$$P_i = V_{CC}\ I_{dc} = 27.3\ \text{W}$$

$$\eta = V_{eff}/V_{CC} = 91.7\% \quad .$$

14-4.1 The purpose of the third-harmonic component is to flatten the collector voltage waveform, thereby increasing the allowable output, hence the efficiency and output capability as well. The ratio of V_{cm3} to V_{om} can therefore be determined by regarding V_{om} as constant and varying V_{cm3} to produce the minimum peak-to-peak variation of

$$v_C(\theta) = V_{CC} + V_{om} \sin\theta + V_{cm3} \sin 3\theta \ .$$

The first two derivatives are

$$\frac{\partial v_c(\theta)}{\partial\theta} = V_{om} \cos\theta + 3\ V_{cm3} \cos 3\theta$$

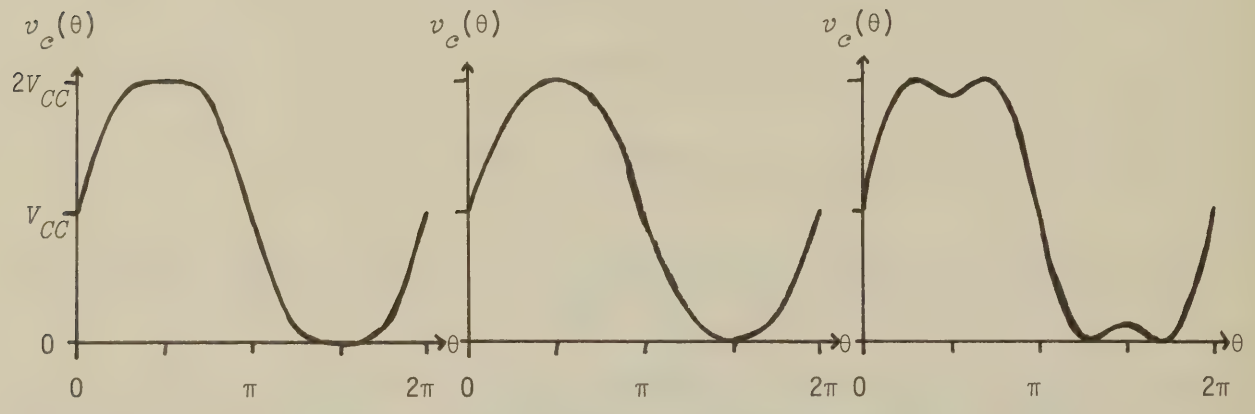
and

$$\frac{\partial^2 v_c(\theta)}{\partial\theta^2} = -\ V_{om} \sin\theta - 9\ V_{cm3} \sin 3\theta \ .$$

Zeros of the first derivative indicate inflection points; it is evident that $\theta = \pi/2$ is an inflection point. Examination of the second derivative at that inflection point shows

$$\left.\frac{\partial^2 v(\theta)}{\partial \theta^2}\right|_{\theta = \pi/2} \begin{cases} > 0, & V_{cm} < 9\, V_{cm3} \quad (v_c(\theta) \text{ minimum}) \\ < 0, & V_{cm} > 9\, V_{cm3} \quad (v_c(\theta) \text{ maximum}) \end{cases}$$

Setting $V_{cm3} = V_{om}/9$ will produce a zero second derivative, hence a saddle point and maximum flatness in $v_c(\theta)$.



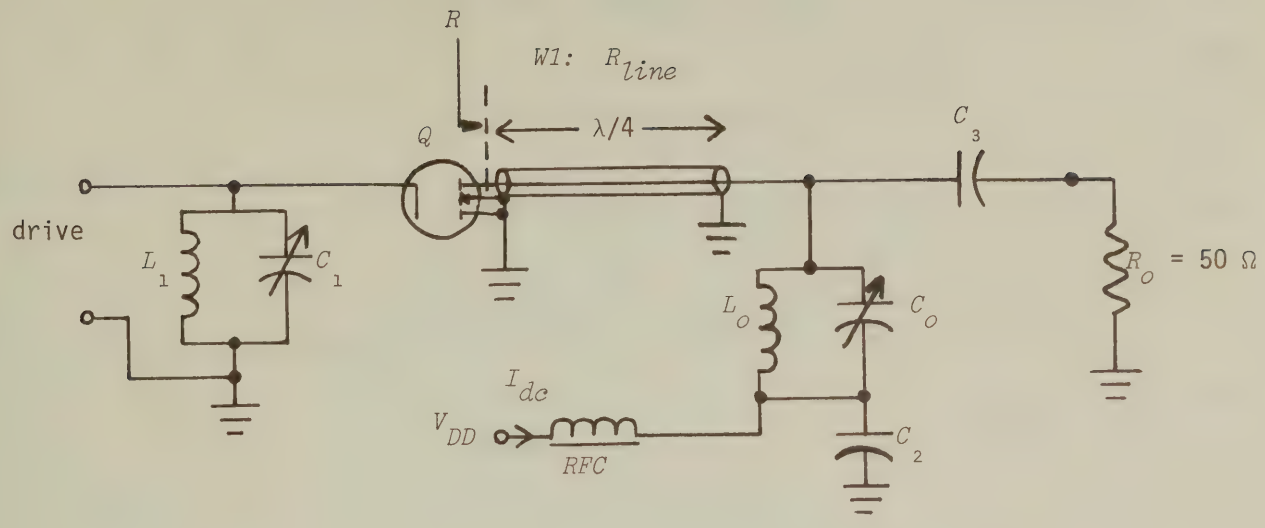
14-4.2 By using the equations in Appendix 14-1 (which ignore R_{on}) the drain load resistance must be

$$R \leq \frac{8}{\pi^2}\,\frac{9^2}{0.250} = 263\ \Omega\ .$$

Since an available "standard-value" transmission line has a characteristic impedance R_{line} = 95 Ω,

$$R = 95^2/50 = 180.5\ \Omega\ .$$

Since the drain load resistance is higher than the 50-ohm load impedance, it is preferable to feed dc at the load end of the line so that a smaller choke RFC can be used. The circuit then has the form:



For Q = 3, $X_{L_o} = X_{C_o}$ = 50/3 = 16.6 ohms, hence for f = 121.5 MHz, L_o = 21.9 nH and C_o = 78.9 pF. For $X_{C_2} = X_{C_3} \leq$ 50/100 ohms, $C_2 = C_3 \geq$ 0.0026 μF and a parallel combination of 0.005 μF, 470 pF, and 27 pF might be used. For X_{RFC} = 50·100 ohms, L_{RFC} = 6.5 μH. The line is a quarter wavelenght long, hence with a velocity of about 0.95 c,

$$l_{line} = \frac{0.95\ c}{4f} \cong 58.6 \text{ cm.}$$

The input tuned circuit is not critical and can have the same values as the output circuit or can be designed to suit the driver.

The effective supply voltage for a 250 mW output is found by using the formula in Appendix 14-1;

$$V_{eff} = \sqrt{\frac{\pi^2}{8} R P_o} = \sqrt{\left(\frac{\pi^2}{8}\right) (180.5)(0.25)} = 7.46 \text{ V} \quad .$$

The actual supply voltage V_{DD} is found from (14-31):

$$V_{DD} = \frac{R + 2 R_{on}}{R} V_{eff} = \left(\frac{180.5 + 30}{180.5}\right) (7.46) = 8.70 \text{ V} \ .$$

Since the efficiency is

$$\eta = \frac{R}{R + 2 R_{on}} = 85.7\% \ ,$$

$$P_i = P_o/0.857 = 292 \text{ mW}$$

and

$$I_{dc} = P_i/V_{DD} = 33.5 \text{ mA}.$$

The second-to-last column of Appendix 14-1 then gives

$$i_{d,max} = I_{dm} = \pi I_{dc} = 105.2 \text{ mA}.$$

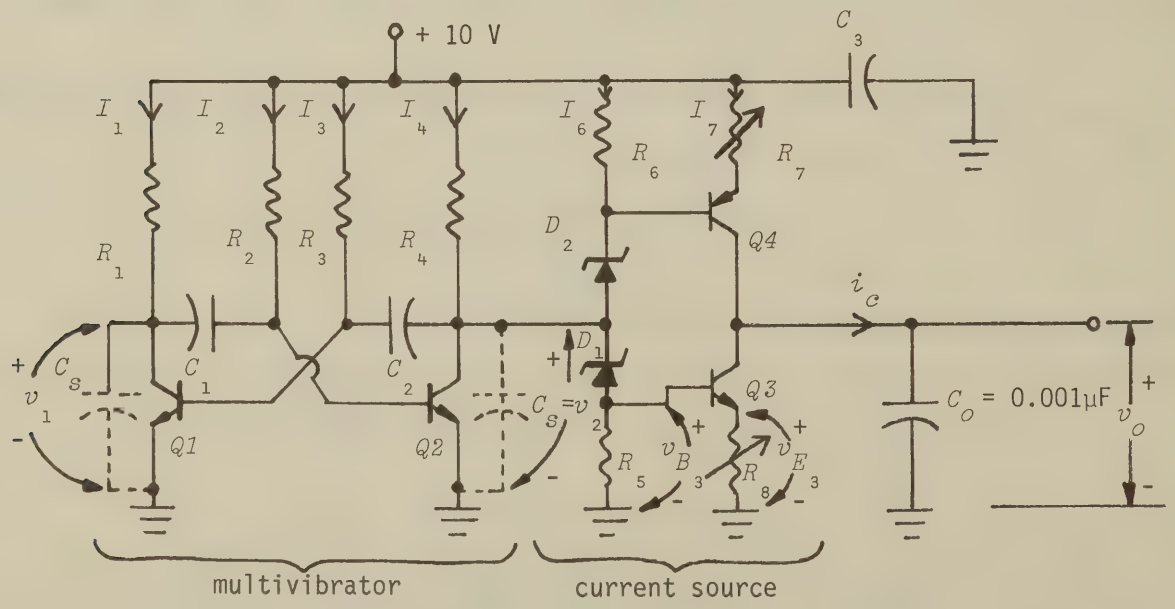
The voltage rating is

$$v_{d,max} = 2 V_{DD} = 17.4 \text{ V}.$$

The required peak driving voltage is

$$V_{gsm} = I_{dm}/g_m = 10.52 \text{ V} .$$

14-5.1 A suitable circuit arrangement is shown below. The multivibrator is a standard configuration, and the current source allows for both charging and discharging of the output capacitance.



First, design the multivibrator. Since T = 10 μsec = 1/100 kHz, we must have $(R_1)\ C_s \ll$ 10 μsec for rapid pull-up of the collector voltage on *Q1*, hence

$$R_1 \ll 10\ \mu\text{sec}/100\ \text{pF} = 100\ \text{K},$$

and we select

$$R_1 = R_4 = 1\ \text{K}.$$

With reasonable current gain for *Q1* and *Q2*,

$$R_2 = R_3 = 10\ \text{K}.$$

Now

$$V_{CC} - V_{sat} - V_\gamma = 10 - 0.3 - 0.7 = 9\ \text{V},$$

hence the recharging collector voltage waveforms have the form

$$v_1 = 10 - 19\ \exp[-t/(R_2 \cdot C_1)] \quad .$$

Setting $v_1 = V_\gamma$ and $t = T/2$ to correspond to one half cycle at 100 kHz produces

$$-9.3 = -19\ \exp[-T/(2 \cdot R_2 \cdot C_1)]$$

$$\exp[T/(2 \cdot R_2 \cdot C_1)] = 19/9.3 = 2.04$$

$$T/(2 \cdot R_2 \cdot C_1) = 0.714$$

$$C_2 = C_1 = \frac{10^{-5}}{2 \cdot 10^4 \cdot 0.714} = 700\ \text{pF}.$$

Voltage v_2 is now a 100 kHz squarewave with levels of 0.3 and 10.0 V. Current-source operation is achieved because transistors *Q3* and *Q4* produce enough collector current to keep their emitter voltages V_γ above or below their base voltages. For example, the collector current of *Q3* is $(v_{B3} - V_\gamma)/R_8$. The Zener diodes ensure that either *Q3* or

$Q4$ will be cut off at any given time and also allow for lower voltage drops across the emitter resistors R_7 and R_8.

The current required to charge or discharge C_o to ± 4 V from its previous value twice each 10 μsec is

$$i_c = \frac{\Delta v_o \, C_o}{T/2} = \frac{4 \cdot 10^{-9}}{10^{-5}/2} = 0.8 \text{ mA} .$$

A standard Zener diode voltage is 6.3 V. Therefore, when $Q3$ is on ($v_2 = 10$ V),

$$v_{B_3} = v_2 - v_{D_1} = 10 - 6.3 = 3.7 \text{ V},$$

$$v_{E_3} = v_{B_3} - V_\gamma = 3.7 - 0.7 = 3.0 \text{ V},$$

hence

$$R_8 = \frac{v_{E_3}}{i_C} = \frac{3.0}{0.8} = 3.75 \text{ K} .$$

Similarly, when $Q4$ is on, $v_2 = 0.3$ V,

$$v_{B_4} = v_2 + v_{D_2} = 6.6 \text{ V},$$

and

$$v_{E_4} = v_{B_4} + V_\gamma = 7.3 \text{ V},$$

hence

$$R_7 = (10 - 7.3)/0.0008 = 3.38 \text{ K}.$$

Since R_7 and R_8 determine the value of the charging current i_c, they must be either precision-valued or adjustable. The values of R_6 and R_5 are not critical and should be about 5 times the values of R_7 and R_8, here

$$R_5 = R_6 = 12 \text{ K}.$$

Current comsumption in the multivibrator can be determined as follows:

$$I_1 + I_4 = (10 - 0.3)/1000 = 9.30 \text{ mA},$$

$$I_2 + I_3 \cong [10 - (+ 0.7 - 9)/2]/10^4 = 1.42 \text{ mA}.$$

Current flows in R_6 and R_7 only half of the time, hence

$$I_7 = \tfrac{1}{2}\ (0.8) = 0.40 \text{ mA},$$

$$I_6 = \tfrac{1}{2}\ (10 - 6.6)/33.8 \text{ K} = 0.05 \text{ mA}.$$

The total current and power consumption is

$$I_{dc} = 9.3 + 1.42 + 0.4 + 0.5 = 11.6 \text{ mA}$$

$$P = 10\ I_{dc} = 116 \text{ mA};$$

note most of the power is consumed in the multivibrator.

By-pass capacitor C_3 should have

$$X_{C_3} \leq (V_{CC}/I_{dc})/100 = 16.67\ \Omega\ ,$$

at 100 kHz, hence $C_3 \geq 9.55 \cdot 10^{-8}$ F and we select

$$C_3 = 0.1\ \mu\text{F}.$$

14-5.2 The purpose of this small design project is to design the modulator for the marine SSB transmitter of Problem 14-2.3. The pulse amplifiers will be arranged in complementary pairs for rapid switching and will use Zener diodes to interface different voltage levels.

First, *Q1* and D_1 must have ratings of at least 30 V and 6.42 A. Now for saturation of *Q1*,

$$i_{b_1} \geq i_{C_1}/\beta = 6.42/15 = 428 \text{ mA}.$$

(A Zener diode should not be used here to step down the voltage because the power dissipation is excessive - 428 mW/V.) Saturation of *Q1* is ensured at minimum V_{CC} = 26 V by

$$R_1 \leqq \frac{V_{CC} - V_{\gamma(Q1)} - V_{sat(Q3)}}{i_{b_1}} = \frac{26 - 0.7 - 0.5}{0.428} = 57.9\ \Omega\ ,$$

hence

$$R_1 = 56\ \Omega.$$

At a maximum V_{CC} = 30 V,

$$i_b = \frac{30 - 0.7 - 0.5}{56} = 514\ \text{mA}\ ,$$

hence

$$P_{R_1} = (30 - 0.7 - 0.5)(0.514) = 14.8\ \text{W}.$$

(This might tempt one to use a Darlington arrangement in place of *Q1* to reduce power dissipation. However, the full 14.8 W is dissipated only at the full modulator output power of 160 W.)

Saturation of *Q1* at maximum V_{CC} is ensured by allowing for i_{C_3} = 514 mA, hence

$$i_{b_3} \geqq i_{C_3}/\beta = 514/15 = 36.1\ \text{mA}\ ,$$

hence

$$R_5 \leqq \frac{12 - 0.7 - 0.5}{0.0361} = 300\ \Omega\ .$$

We choose the standard value

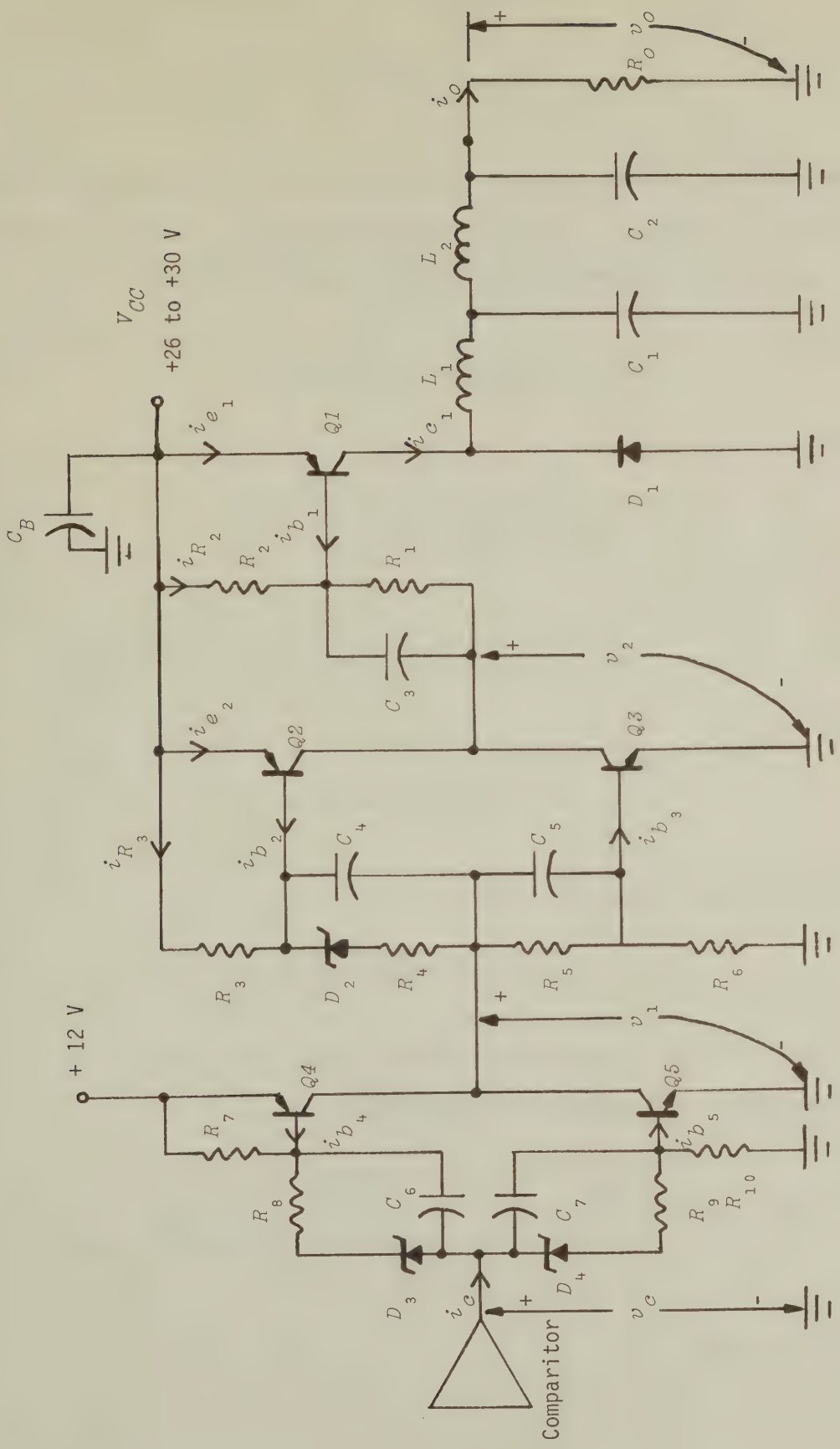
$$R_5 = 270\ \Omega\ ,$$

hence

$$i_{b_3} = \frac{12 - 0.7 - 0.5}{270} = 40\ \text{mA}.$$

To span the 26 to 30 V range requires for *Q2* cutoff and saturated, respectively:

$$V_{CC} - V_{sat} + V_{D_2} \geqq 30 - V_{\gamma},\ \text{hence}\ V_{D_2} > 17.8\ \text{V}$$


V_{CC}
+26 to +30 V
+ 12 V
C_B
i_{e_1}
Q1
i_{c_1}
L_1
L_2
C_1
C_2
i_O
R_O
v_O
i_{R_2}
R_2
i_{b_1}
R_1
C_3
v_2
i_{e_2}
Q2
Q3
i_{R_3}
i_{b_2}
C_4
C_5
i_{b_3}
R_3
D_2
R_4
R_5
R_6
v_1
Q4
Q5
R_7
i_{b_4}
i_{b_5}
R_8
C_6
C_7
R_9
R_{10}
D_3
D_4
i_e
v_e
Comparitor

$$V_{sat} + V_{D_2} < 26 - V_\gamma, \text{ hence } V_{D_2} < 24.8 \text{ V}.$$

A suitable choice is $V_{D_2} = 20.2$, hence D_2 is really two 10.1 V Zener diodes in series. For rapid pull-up of voltage v_2, i_{c_2} should be comparable to i_{c_3}, hence i_{b_2} should be comparable to i_{b_3}. To keep the power dissipation per diode below 400 mW,

$$i_{b_2} \leq 400 \text{ mW}/10.1 \text{ V} = 39.6 \text{ mA}.$$

Therefore with $V_{CC} = 30$ V,

$$R_4 \geq \frac{30 - 0.7 - 0.5}{0.0396} = 727 \ \Omega \ ,$$

hence a standard value

$$R_4 = 750 \ \Omega \ ,$$

is chosen and

$$i_{b_2} \leq \frac{30 - 0.7 - 0.5}{750} = 38.4 \text{ mA} \ .$$

To ensure saturation of *Q4*,

$$i_{b_4} \geq i_{c_4}/\beta = i_{b_3}/\beta = 40/15 = 2.67 \text{ mA};$$

since this is less than the 3 mA available from the comparater (and $i_{b_5} \cong i_{b_4}$), additional stages should be needed. To span the voltage differences,

$$9 + v_{D_3} \geq 12 - 0.7, \text{ hence } v_{D_3} > 2.3$$

$$6 + v_{D_3} \leq 12 - 0.7, \text{ hence } v_{D_3} < 5.3 \ .$$

Therefore a standard value is

$$v_{D_3} = 3.3 \text{ V.}$$

Then

$$R_8 \leq \frac{(12 - 0.7) - (6 + 3.3)}{2.67 \cdot 10^{-3}} = 749\ \Omega \quad ,$$

hence

$$R_8 = 680\ \Omega \text{ (a standard value)}$$

and

$$i_{b_4} = \frac{(12 - 0.7) - (6 + 3.3)}{680} = 2.94 \text{ mA} \quad .$$

Similarly,

$$0.7 + v_{D_4} < 9, \text{ hence } v_{D_4} < 8.3 \text{ V} \quad ,$$

$$6 - v_{D_4} < 0.7, \text{ hence } v_{D_4} > 5.3 \text{ V,}$$

hence

$$v_{D_4} = 6.3 \text{ V (standard value).}$$

With

$$R_9 = 680\ \Omega \ ,$$

$$i_{b_5} = \frac{9 - 0.7 - 6.3}{680} = 2.94 \text{ mA.}$$

The values of R_2, R_3, R_6, R_9, and R_{10} are noncritical and should be about 5 to 10 times the nearby current-carring resistor, thus

$$R_2 = 330\ \Omega$$

$$R_6 = R_3 = 3.3 \text{ K}$$

$$R_7 = R_{10} = 4.7 \text{ K}$$

Values of C_3, C_4, C_5, C_6, and C_7 are determined empirically in the absence of C_{ob} data and R_B data.

The specified 80-dB attenuation in a single decade requires a four-pole (two L-section) filter. To get a particular roll-off characteristic, it is necessary to consult a filter design manual. If the particular roll-off characteristic is not important, one can use two L-sections that would have $\zeta = 0.707$ if they were loaded by resistances. From the information in the text,

$$\sqrt{L_1 C_1} = \sqrt{L_2 C_2} = 1/\omega_{cutoff} = 1/2\pi \cdot 2 \cdot 10^4 = 7.96 \cdot 10^{-6}$$

$$\sqrt{L_1/C_1} = \sqrt{L_2/C_2} = 16.$$

Thus

$$L_1 = L_2 = 16(7.96 \cdot 10^{-6}) = 127.3 \ \mu\text{H} \quad ,$$

and

$$C_1 = C_2 = 0.497 \ \mu\text{F}.$$

The shunt capacitance of 100 pF is charged by V_{eff} once per switching cycle, thus

$$P_s = (C \ V_{eff}^2) \ f = (2 \cdot 10^5)(10^{-10}) V_{eff}^2$$

$$= 2 \cdot 10^{-5} \ V_{eff}^2 \leq 18 \text{ mW} \ .$$

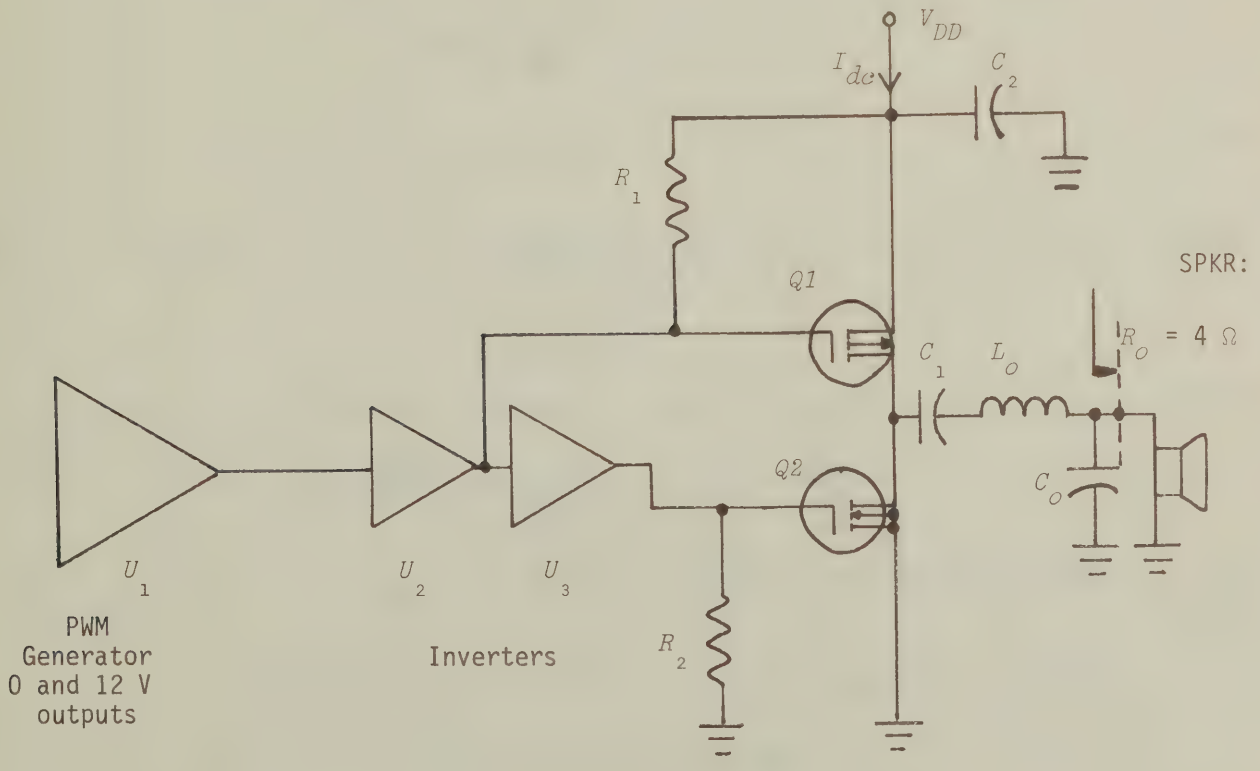
and can therefore be ignored. The power consumed in each cycle is $C \ V_{eff}^2$. Charging C_s dissipates $\frac{1}{2} \ C_s \ V_{eff}^2$ and stores $\frac{1}{2} \ C_s \ V_{eff}^2$; the stored $\frac{1}{2} \ C_s \ V_{eff}^2$ is dissipated when C_s is discharged. The currents required by R_3, $Q2$, and R_2 can also be ignored in comparison to i_{e_1}. Since the average current is proportional to the duty ratio,

$$I_{dc} = (i_{c_1} + i_{b_1})(V_o/V_{eff}) = 6.93(V_o/V_{eff}) \text{ A} \ ;$$

the results are shown in the table for maximum output (V_o = 25 V, P_o = 160.5 W).

V_{CC}	26	30
V_o/V_{eff}	0.980	0.847
I_{dc}	6.79 A	5.87 A
P_i	176.6 W	176.2 W
η	90.9%	91.1%

14-5.3 Considerable circuit simplification is possible with FETs, since the required driving power is negligible. If the swithcing frequency is low enough, complementary FETs can be used and CMOS circuitry can be used as the driver, as shown in the circuit diagram below:



For P_o = 2.5 W into R_o = 4 ohms,

$$V_{om} = \sqrt{2 \cdot 2.5 \cdot 4} = 4.47 \text{ V}$$

and

$$I_{om} = V_{om}/R_o = 1.12 \text{ A} .$$

The "voltage drop" across R_{on} is

$$V_{on} = R_{on} I_{on} = 0.56 \text{ V},$$

hence it is necessary that

$$V_{DD} > 2(V_{om} + V_{on}) = 10.06 \text{ V},$$

for full output. Since this is the case, no impedance transformation is necessaary.

Since one R_{on} is always in series with R_o, the efficiency is

$$\eta = \frac{R_o}{R_o + R_{on}} = \frac{4}{4.5} = 88.9\% ,$$

and

$$P_i = P_o/\eta = 2.813 \text{ W},$$

hence

$$\frac{2.813}{12} = 0.234 \leq I_{dc} \leq 0.201 = \frac{2.813}{14} \text{ A} ,$$

if R_1 is ignored. The peak ratings for *Q1* and *Q2* are $v_{D,max}$ = 14 V and $i_{D,max} = I_{om}$ = 1.12 A.

With g_m = 0.1 mho,

$$v_{gs(on)} \geq I_{om}/g_m = 11.2 \text{ V},$$

which is just right for drive from CMOS circuitry. Resistors R_1 and R_2 have values in the range of 100 K and serve only to prevent any static charge build-up on the gates of *Q1* and *Q2*. Inverters U_2 and U_3 might be made from CMOS or gates. Circuit U_1 is the comparator circuit of a PWM generator, which might be made by biasing a CMOS gate into the transition region so that a slight difference between the input audio and triangular signals would produce a high or low output.

The highest modulation depth necessary occurs with V_{DD} = 12 V, hence

$$\frac{(V_{om} + V_{on})}{V_{DD}} \leq \frac{4.47 + 1.12\ (0.5)}{12} = 0.419\ .$$

By using this number in Figure 14-15, one can determine that $f_s \geq 6\ f_m$ will keep the f_s - 4 f_m and all other spurious products below the desired -40 dB level. Since 18 kHz is an inconveniently low frequency (L_o, C_o, C_1, and C_2 would have inconveniently large values), we select f_s = 50 kHz. As in example 14-5.2,

$$\sqrt{L_o\ C_o} = 1/(2\pi \cdot 3 \cdot 10^3) = 5.31 \cdot 10^{-5}$$

$$\sqrt{L_o/C_o} = 16$$

$$L_o = 849\ \mu H$$

$$C_o = 3.33\ \mu F,$$

For $X_{C_1} \leq$ 4/10 ohms at f = 300 Hz (a reasonable low-frequency cutoff), $C_1 \geq$ 1326 μF, hence

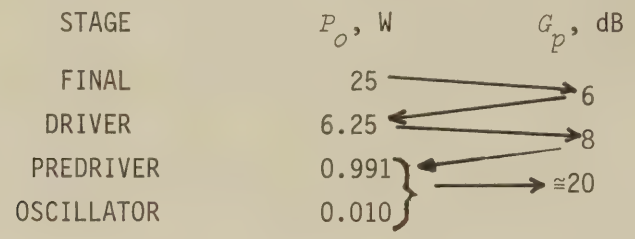
$$C_1 = 1500\ \mu F.$$

For $X_{C_2} \leq$ (12 V/1.12 A)/100 at f_s = 50 kHz, $C_2 \geq$ 29.7 μF, hence

$$C_2 = 50\ \mu F + 0.005\ \mu F.$$

CHAPTER 15 PROBLEMS

15-1.1 It is usually convenient to "rough out" a chain design (or part of it) by working from power to power gain to drive power. Figure 15-1 is used for this problem, with arrows used below to indicate the order of computation:



At this point, it is apparent that about 20 dB more gain is needed, which can be divided into two stages. We how prepare a formal power, gain, and efficiency table:

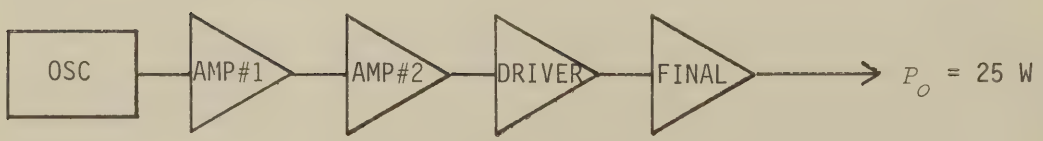
STAGE	CLASS	P_o, W	G_p, dB	η	P_i, W
OSC	-	0.01	-	0.3	0.033
AMP#1	A	0.10	10	0.3	0.333
AMP#2	C m-m	0.991	9.96	0.6	1.667
DRIVER	C m-m	6.25	8	0.6	10.417
FINAL	C m-m	25.0	6	0.6	41.667

From the above,

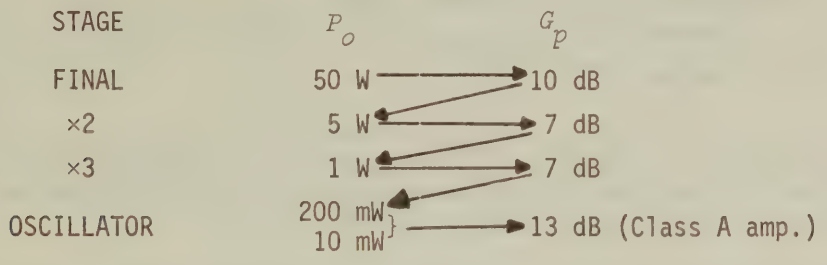
$$P_{i,TOTAL} = 54.116 \text{ W}$$

$$\eta = 25/54.116 = 46.2\%$$

$$I_{dc} = P_i/V_{CC} = 54.116/12 = 4.5 \text{ A.}$$



15-1.2 Frequency must be multiplied by a factor of (8.33 × 6 = 50), hence two frequency-multiplying stages are required. The frequency doubler should follow the frequency trippler to provide maximum reduction of harmonics in the output signal. Again, roughing-out the chain is a convenient way to start:



Now prepare the formal table:

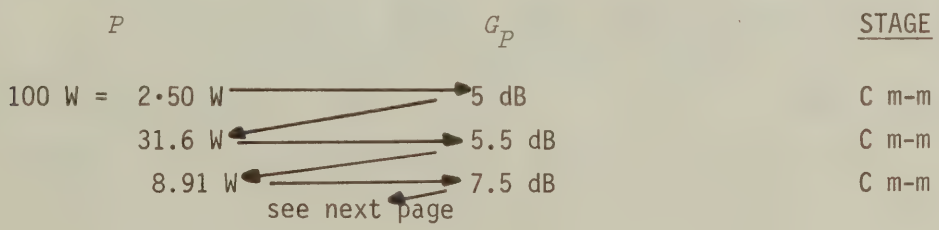
STAGE	CLASS	OUTPUT FREQUENCY MHz	P_o, W	G_p, dB	η	P_i, W
OSC	-	8.33	0.01	-	0.3	0.033
AMP	A	8.33	0.20	13	0.3	0.667
×2	C m-m	25	1	7	0.6	1.667
×3	C m-m	50	5	7	0.6	8.333
AMP	C m-m	50	50	10	0.6	83.333

$$P_{i,TOTAL} = 94.034 \text{ W}$$

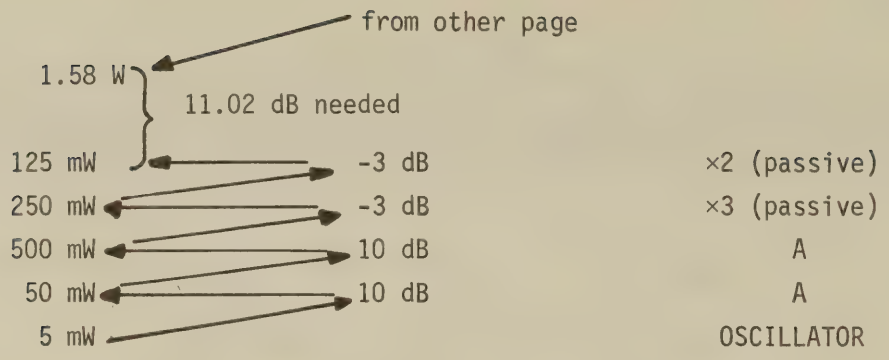
$$\eta = 50/94 = 53.2\%$$

$$I_{dc} = P_i/V_{CC} = 7.84 \text{ A.}$$

15-1.3 This chain can be "roughed out" by working from both ends toward the middle (using Figure 15-1 to determine G_p):



P15-1.3



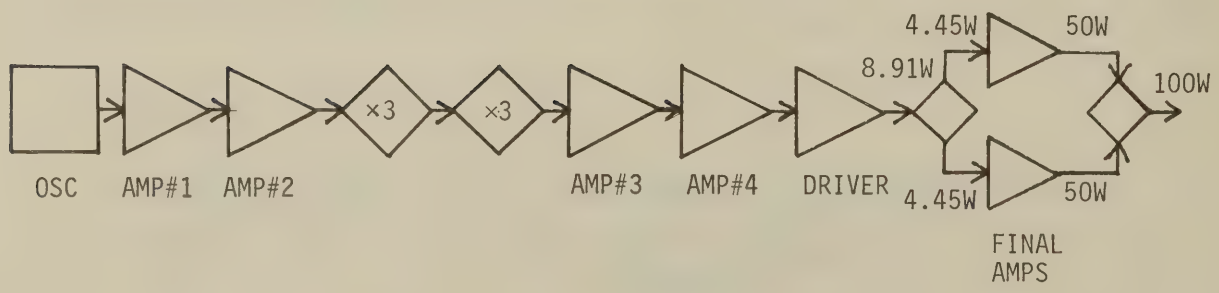
We decide to obtain 10 of the 11.02 dB gain needed from a Class C mixed-mode state; the remaining 1.02 dB will be obtained by increasing the gain of a low-power Class A amplifier. The formal gain-and-power table is then:

STAGE	CLASS	FREQUENCY MHz	G_p dB	P_o W	η	P_i W
Oscillator	-	8.33	-	0.005	0.3	0.017
AMP#1	A	8.33	11	0.063	0.3	0.210
AMP#2	A	8.33	10	0.630	0.3	2.100
Diode mult, ×3	-	25	-3	0.315	-	-
Diode mult, ×2	-	50	-3	0.157	-	-
AMP#3	C m-m	50	10	1.58	0.6	2.633
AMP#4	C m-m	50	7.5	8.91	0.6	14.850
Driver Amp	C m-m	50	5.5	31.6	0.6	52.700
Final Amps	C m-m	50	5	100	0.6	166.667

$$P_{i,TOTAL} = 239.2 \text{ W}$$

$$\eta_{XMTR} = 100/239.2 = 41.8\%$$

$$I_{dc} = P_{i,TOTAL}/12 \text{ V} = 19.9 \text{ A}$$



15-2.1 A square-law device has the characteristics

$$v_o = (V_i + v_i)^2 = V_i^2 + 2\,V_i\,v_i + v_i^2$$

where v_o and v_i are the instantaneous output and input voltages, respectively, and V_i is a dc bias voltage or circuit constant,

(a) For frequency-doubling of an FM signal,

$$v_i = A \sin [\omega t + \theta(t)] ,$$

where A is constant and $\theta(t)$ represents the phase variation due to modulation. The output signal is then

$$v_o = V_i^2 + 2\,V_i\,A \sin [\omega t + \theta(t)] + A^2 \sin^2 [\omega t + \theta(t)]$$

$$= (V_i^2 + \tfrac{1}{2} A^2) + 2\,V_i\,A\,[\sin \omega t + \theta(t)]$$

$$- \tfrac{1}{2} A^2 \cos [2\,\omega t + 2\,\theta(t)].$$

The phase variation in the double-frequency component of v_o is $2\,\theta(t)$, twice that $[\theta(t)]$ in the input signal. The modulation index and frequency deviation are therefore also doubled, since the modulating frequency is unchanged.

(b) For mixing,

$$v_i = A_1 \sin [\omega_1 t + \theta(t)] + A_2 \sin \omega_2 t \quad .$$

The output is then

$$v_o = V_i^2 + 2\,V_i\,A_1 \sin [\omega_1 t + \theta(t)] + 2\,V_i\,A_2 \sin \omega_2 t$$
$$+ A_1^2 \sin^2 [\omega_1 t + \theta(t)]$$
$$+ A_1\,A_2 \sin [\omega_1 t + \theta(t)] \sin \omega_2 t + A_2^2 \sin^2 \omega_2 t$$

$$= (V_i^2 + \tfrac{1}{2} A_1^2 + \tfrac{1}{2} A_2^2) + 2\,V_i\,A_1 \sin [\omega_1 t + \theta(t)] + 2\,V_i\,A_2 \sin \omega_2 t$$

$$+ \tfrac{1}{2} A_1 A_2 \cos [(\omega_1 - \omega_2)t + \theta(t)]$$

$$- \tfrac{1}{2} A_1 A_2 \cos [(\omega_1 + \omega_2)t + \theta(t)] \quad .$$

$$-\tfrac{1}{2} A_1^2 \cos[2\,\omega t + 2\,\theta(t)] - \tfrac{1}{2} A_2^2 \cos 2\,\omega_2 t$$

It is evident that the phase variation, hence frequency deviation and modulation index of the signals of frequency $\omega_1 - \omega_2$ and $\omega_1 + \omega_2$ are the same as those of the input signal.

15-2.2 The circuit of Figure 15-4 is applicable to this problem; remember that we are designing only the coupling network in this problem and *not* the entire oscillator circuit. This problem requires attention to the discussion in the text and recognizing approximately equal values of capacitance.

A convenient "quiescent" diode voltage is V_D = -6 V = $-\frac{1}{2} V_{CC}$, hence (15-2) produces

$$C_{VVC} = \frac{C_{-1V}}{(-V_D)^N} = \frac{150\ \text{pF}}{6^{\frac{1}{2}}} = 61.2\ \text{pF}.$$

From the requirements,

$$\frac{\Delta f/36}{f/36} = \frac{\Delta f}{f} = \frac{1\ \text{kHz}}{52\ \text{MHz}} = 1.92 \cdot 10^{-5} \quad .$$

To prevent loading of the crystal but allow (possibly) for grounding the "low" sides of the crystal and diode, set

$$C_1 = 10\ \text{pF}$$

$$C_2 = 0.01\ \mu\text{F},$$

hence

$$C_{12} \cong 10\ \text{pF}$$

and

$$C_N \cong \frac{1}{1/61.2 + 1/10} \cong 8.60\ \text{pF}.$$

Now rearrange (15-5) and insert values of C_p, C_s, C_N, C_{12}, C_{VVC}, and $\Delta f/f$:

$$\frac{\Delta v_D}{V_D} = \frac{2}{N} \frac{(C_p + C_N)^2\ C_{VVC}}{C_s\ C_{12}^2} \frac{\Delta f}{f}$$

$$= \frac{2}{\frac{1}{2}} \frac{(12.6^2)(61.2)}{(0.04)(10^2)} (1.92 \cdot 10^{-5}) = 0.187 \quad ,$$

hence

$$|\Delta v_D| = 0.187 \ |V_D| = 1.12 \text{ V.}$$

To check the approximations and to estimate the linearity, determine the resonant frequency for maximum and minimum $v_D(t)$:

v_D, V	C_{VVC}, pF	C_N	$C_p + C_N$	C_x	$\sqrt{C_x/C_{x,6V}}$	f	Δf, Hz
6 - 1.12 = 4.88	67.902	8.716	12.716	0.0398746	1.0000149	52.000777	+777
6	61.237	8.596	12.596	0.0398734	1.0000000	52.0	0
6 + 1.12 = 7.12	56.215	8.490	12.490	0.0398723	0.9999865	51.999298	-702

Distortion $\cong \frac{777 - 702}{777 + 702} = 5.07\%$,

which is acceptable for speech communications. From the above table, it appears that the actual voltage deviation for a 1 kHz frequency deviation must be.

$$\Delta v_D \cong (2 \cdot 1000)/(777 + 702) = 1.51 \text{ V.}$$

Nonetheless, the equation given are useful for a rough design.

The chokes must have a relatively low impedance to AF signals and a relatively high impedance to RF signals, thus

$$X_{RFC} << X_{VVC} \text{ at } f = 3 \text{ kHz}$$

$$X_{RFC} >> X_{VVC} \text{ at } f = 1.44 \text{ MHz} ;$$

the second condition is more important, since it affects operation of the oscillator. For $X_{RFC} = 10 \ X_{VVC} = 180.6 \text{ k}\Omega$ at $f = 1.44$ MHz,

$$L_{RFC} = 20 \text{ mH.}$$

At $f = 3$ kHz, this produces $X_{RFC} = 376 \ \Omega = X_{VVC}/2304$.

15-2.3 Set (by design) the "quiescent" or zero-signal phase shift $\theta_0 = -20°$ so that

exceeding $\Delta\theta = \pm 10°$ does not abruptly produce clipping of the modulating signal. Similarly, set the quiescent input voltage at $V_{gs} = -6$ V so that $g_{fs} = 6 \cdot 10^{-4}$ (i.e., the FET is similar to a 1.67 $k\Omega$ resistor when there is no AF input signal). Therefore, $R_s = 470\ \Omega$ (not critical) and $C_s = 0.005\ \mu F$ so that $X_{C_s} \ll 470\ \Omega$.

By rearrangement of (15-9),

$$x^2 - \frac{2}{\tan \theta} x - 1 = 0 \quad ,$$

where

$$x = \frac{g_{fs}}{\omega C} \quad .$$

Substitution of $\theta = \theta_0 = -20°$ produces

$$x = 0.176$$

hence

$$C = 62.6\ pF.$$

Now we calculate the true performance of the circuit from (15-9) by remembering that g_{fs} is assumed to be proportional to V_{gs}:

V_{gs}, V	θ (desired)	θ (actual)
-3	-10°	-10.06°
-6	-20°	-20°
-9	-30°	-29.6°

The deviation from linearity is no more then 4% (0.4° in 10°) which is roughly equivalent to -28 dB. This is acceptable for speech communication. Full ±10° phase deviation requires a ±3 V peak time-integrated audio signal.

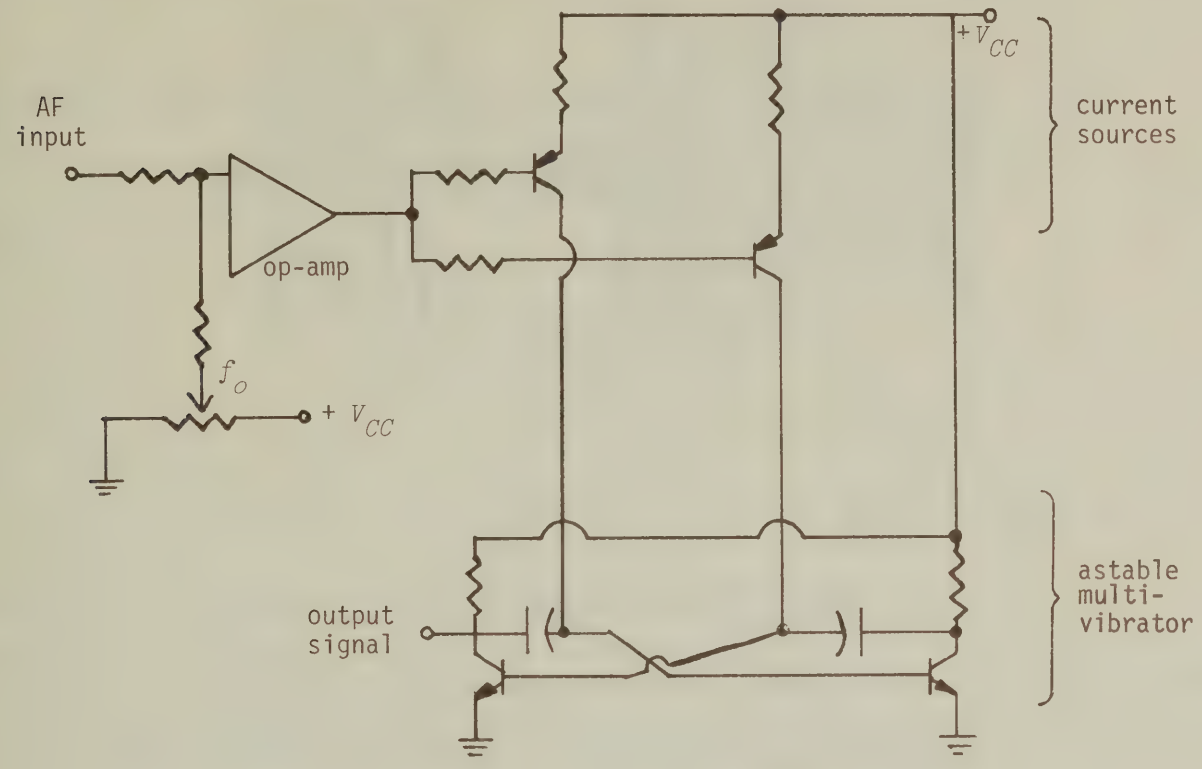
The assumptions used in the analysis in the text require that $R_L \gg 2/g_{fs}$, hence let

$$R_L = 10\ k\Omega$$

$$C_B = 0.001\ \mu F \text{ for } X_{CB} \ll R_L$$

$$L_{RFC} = 2\ mH \text{ for } X_{RFC} \gg R_L \ .$$

15-2.4 A suitable circuit arrangement uses controlled current sources to recharge the timing capacitors in the multivibrator:



Since both charging currents are the same, the symmetry of the output signal squarewave is preserved. Since recharging time is proportional to the recharging current, this circuit actually modulates the *period* of the signal, rather than the frequency. However, for small variations ΔT in the period T,

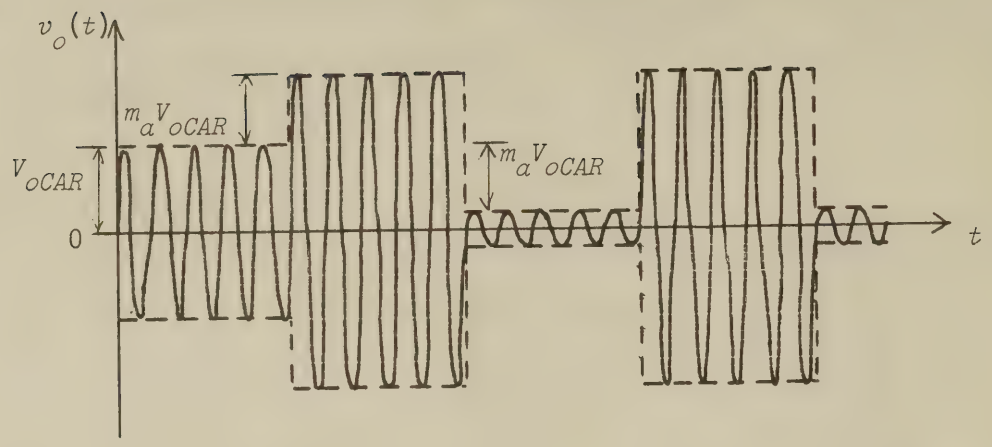
$$\frac{1}{T - \Delta T} \cong f + \Delta f \ .$$

and linear frequency modulation is therefore possible (the deviation from linearity is directly proportional to the frequency deviation; thus a 10 percent frequency deviation has a 10 percent deviation from linearity, etc.).

Disadvantages: The frequency stability of this circuit is not very high, and its use is limited to relatively low frequencies (less than 10 MHz) with current technology.

Advantages: A very large $\Delta f/f$ is attainable, and the circuit is quite simple. Linearity and instability problems can be overcome with AFC and/or FMFB.

15-3.1 (a) The squarewave-modulated carrier waveform is



As for sinusoidal modulation,

$$P_{oPEP} = (1 + m_a)^2 \, P_{oCAR} \leq 4 \, P_{oCAR} \, .$$

However,

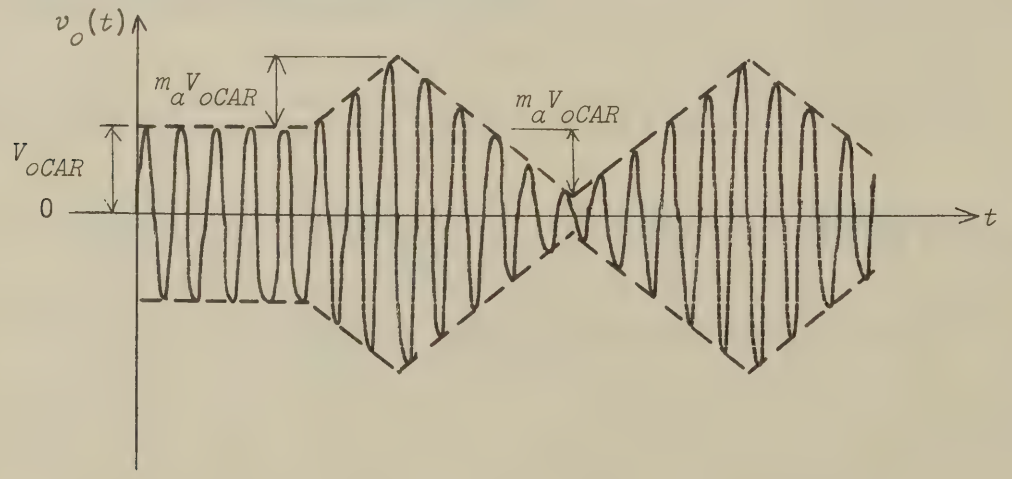
$$P_{oAVG} = \tfrac{1}{2} \{(1 + m_a)^2 \, P_{oCAR} + (1 - m_a)^2 \, P_{oCAR}\}$$

$$= \tfrac{1}{2} \{(2 + 2 \, m_a^2) \, P_{oCAR}\}$$

$$= (1 + m_a^2) \, P_{oCAR} \leq P_{oCAR} \quad ,$$

which is greater than that for sinewave modulation with the same index.

(b) The triangular-wave modulated signal waveform is



As for other modulating waveforms,

$$P_{oPEP} = (1 + m_a)^2 \, P_{oCAR} \leq 4 \, P_{oCAR} \, .$$

The signal envelope is described by

$$V_o(t) = (1 + m_a t/T) \, V_{oCAR}, -T \leq t \leq +T \quad ,$$

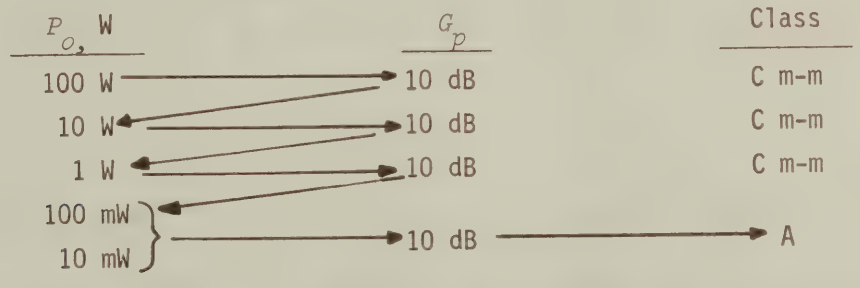
hence

$$P_{oAVG} = \frac{P_{oCAR}}{2T} \int_{-T}^{+T} (1 + \frac{m_a}{T} t)^2 \, dt$$

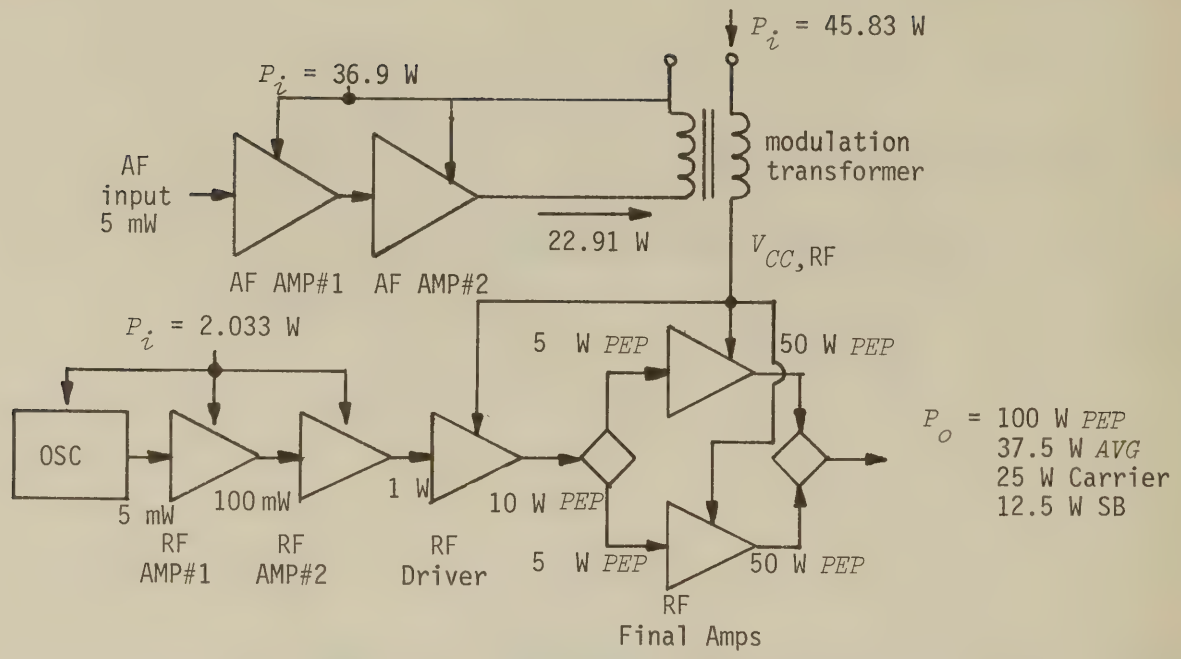
$$= (1 + \frac{1}{3} m_a^2) \, P_{oCAR} \leq \frac{4}{3} \, P_{oCAR} \, ,$$

which is less than that for sinewave modulation with the same index.

15-3.2 A "25 Watt" AM transmitter has a 25-Watt carrier, hence from (15-13), P_{oPEP} = 100 W. Since the power gains are specified exactly, it is a simple matter to assign gains and power levels to the RF stages; capability for peak envelope power must be allowed.



The RF portion of the chain can now be drawn out and the formal gain-and-power table can be made with the information given in this problem. The decision to use power splitters and combiners is entirely arbitrary.



In the absence of information to the contrary, the designer should assume that modulation of the driver is necessary (if the final amplifier uses BJTs). The dc input power to the oscillator and two low-level RF amplifiers is computed as in a CW transmitter. The power input required by the driver and final amplifiers must be split into dc and audio-frequency components using (15-15) and (15-16); remember that $P_{oSB} = P_{oAVG} - P_{oCAR}$. These components are totalled separately in the gain-and-power table, and the latter becomes the required output power of the modulator. The total gain of the audio amplifier chain is

$$G_{p,AF} = 10 \log (22.91/0.005) = 36.6 \text{ dB},$$

which can be accomplished in two stages. A gain-and-power table can be prepared, as shown.

The total power consumed by the transmitter for 100 percent amplitude modulation by a single tone is the sum of the dc input power to the low level RF stages, the dc input power to the AF amplifiers, and the dc input power to the driver and final RF amplifiers that passes through the transmitter secondary winding, hence

$$P_{i,XMTR} = 2.033 + 45.83 + 36.9 = 84.76 \text{ W}.$$

From (15-14), the average output power is

$$P_{oAVG} = 37.5 \text{ W},$$

Stage	Class	P_{oPEP}, W	G_p, dB	η	P_i, W	P_{oCAR}, W	P_{iCAR}, W	P_{oSB}, W	P_{oSB}, W
OSC	-	0.010	-	0.3	0.033	--	--	--	--
RF AMP#1	A	0.10	10	0.3	0.333	--	--	--	--
RF AMP#2	C m-m	1.0	10	0.6	1.667	--	--	--	--
RF Driver	C m-m	10.0	10	0.6	--	2.5	4.167	1.25	2.08
RF Finals	C m-m	100.0	10	0.6	--	25.0	41.667	12.50	20.83

P_i(low-level RF stages) = 2.003 W

P_i(dc to RF driver and final amps) = 45.83 W

P_i(AF power to driver and final amps) = P_o(AF AMP#2) = 22.91 W

Stage	Class	P_o, W	G_p, dB	η	P_i, W
AF Input	-	0.005	--	--	--
AF AMP#1	A	0.500	20	0.35	1.667
AF AMP#2	B	22.91	16.6	0.65	35.231

$P_{iAF,total}$ = 36.9 W

hence

$$\eta_{XMTR} = 44.2\%.$$

15-3.3 A "4-Watt" AM transmitter has P_{oPEP} = 16 W and P_{oAVG} = 6 W for single-tone sinusoidal modulation with m_a = 1.0. Even when the maximum intended modulation index m_a = 0.9, it is a good idea to rate the components for m_a = 1.0 to prevent damage if the audio input exceeds the desired maximum level. Since the output power of the oscillator is 10 mW, the total power gain of the RF chain must therefore be

$$G_{p,chain} = 10 \log (16/0.01) = 32.04 \text{ dB}$$

for peak-envelope-power output.

(a) The 32 dB total power gain is conveniently divided into two stages as shown. Since the driver feed-through ratio is -35 dB and the final amplifier power gain is +16 dB, the feedthrough signal will be 35 + 16 = 51 dB below P_{oPEP}, hence 45 dB below P_{oCAR} and 42 dB below the maximum P_{oSB}. This level is quite satisfactory (remember an *IMD* ratio of -30 dB is often used for linear power amplifiers), and modulation of the driver is unnecessary. The peak and average input power to the final RF amplifier are then determined, and become the required outputs of the Class S modualtor. The average power input to the modulator is then determined by dividing the average output power of the modulator by its efficiency. The total power consumption of the transmitter is then the sum of the input power to the modulator and the two low-level RF stages, hence

$$P_{i,XMTR,AVG} = 0.033 + 1.340 + 12.5 = 13.87 \text{ W}$$

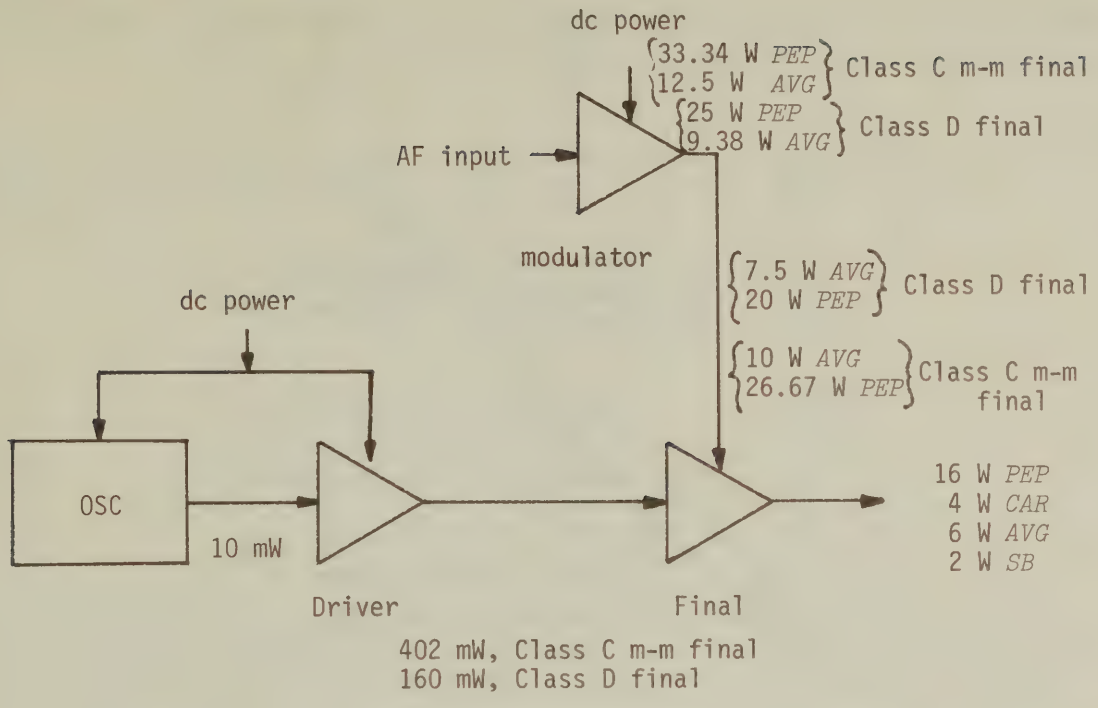
hence

$$\eta_{XMTR,AVG} = 6/13.87 = 43.2\%.$$

Stage	Class	P_{oPEP}, W	η	P_{iPEP}	P_{oAVG}, W	$P_{i,AVG}$, W
OSC	-	0.010	0.3	0.033	-	-
Driver	A	0.402	0.3	1.340	-	-
Final	C m-m	16.0	0.6	26.667	6	10
Mod	S	26.667	0.8	33.34	10	12.5

(b) The design approach and block diagram are similar to those for the Class C mixed-mode final amplifier. Since the Class D final is very efficient, we take advantage of its high gain to minimize the drive power (hence drive feed-through). From the data in the gain-and-power table,

$$P_{i,XMTR,AVG} = 0.033 + 0.533 + 9.375 = 9.941 \text{ W}$$



$$\eta_{XMTR,AVG} = 6/9.941 = 60.4\%.$$

Stage	Class	P_{oPEP}, W	η	P_{iPEP}	P_{oAVG}, W	P_{iAVG}, W
OSC	-	0.010	0.3	0.033	-	-
Driver	A	0.160	0.3	0.533	-	-
Final	D	16.0	0.8	20.0	6.0	7.5
Mod	S	20.0	0.8	25.0	7.5	9.375

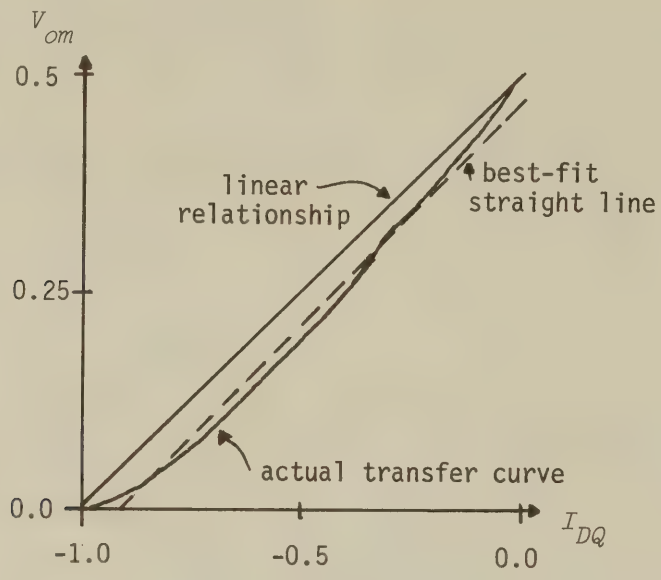
15-3.4 (a) The "quiescent" current I_{DQ} depends upon the gate-bias voltage V_{GS}, threshold voltage V_T and the FET transconductance:

$$I_{DQ} = g_m(V_{GS} - V_T) \quad .$$

The driven component I_{DD} of the drain current is a function of the RF driving voltage and is therefore constant. From (13-3),

$$y = \arccos(-I_{DQ}/I_{DD});$$

equation (13-6) is then used to determine V_{om} as a function of y, hence of I_{DQ}. The graph is based upon R= 1 and I_{DD} = 1.



(b) If R = 1 and I_{DD} = 1 to correspond to the values used in the graph, m_a = 1 and Class B operation at peak envelope power is obtained with

$$I_{DQ} = -\tfrac{1}{2} + \tfrac{1}{2} \sin \theta$$

where $\theta = \omega_m t = 2\pi f_m t$ is defined in terms of the modulating frequency f_m. The procedure given in (a) above is used to compute V_{om} for a given θ. The harmonic components of $V_{om}(\theta)$ are given by

$$a_k = \frac{1}{\pi} \int_0^{2\pi} V_{om}(\theta) \cos k\theta \, d\theta$$

and

$$b_k = \frac{1}{\pi} \int_0^{2\pi} V_{om}(\theta) \sin k\theta \, d\theta \quad .$$

These integrals can be evaluated numerically by simple summations of the form

$$a_k \cong \frac{\Delta\theta}{\pi} \sum_{n=1}^{N} V_{om}(n\Delta\theta)\cos(nk\Delta\theta) \quad ,$$

or more sophisticated techniques such as Simpson's rule; the error is roughly proportional to 1/N. The power P_k at frequency kf_m is then

$$P_k = \tfrac{1}{2}\, a_k^2 = \tfrac{1}{2}\,(a_k^2 + b_k^2).$$

With N = 1000,

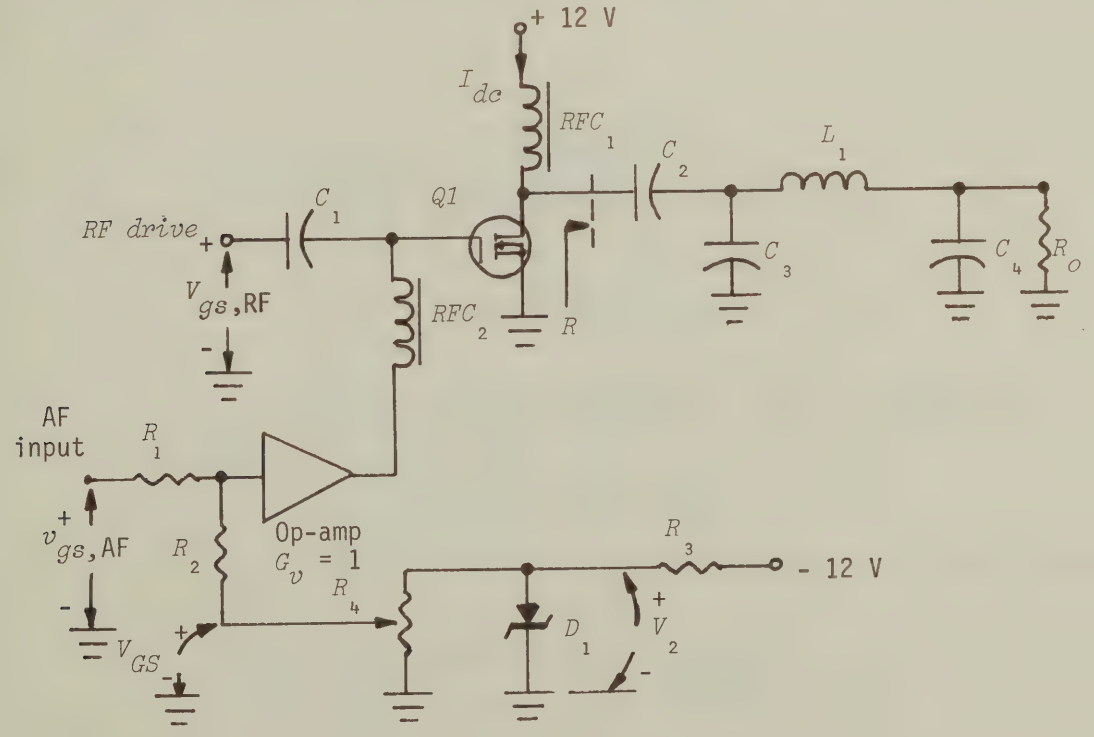
$$a_1 = -5.51\cdot10^{-6} \quad b_1 = 0.257$$

$$a_2 = -2.70\cdot10^{-2} \quad b_2 = -4.88\cdot10^{-6}$$

$$P_2/P_1 = 1.09\cdot10^{-2} \rightarrow -19.6 \text{ dB}.$$

(Essentially the same results were obtained with N = 10). This distortion level will probably be unacceptable in most applications (particularly where adjacent channel interference is a consideration) and audio (envelope) feedback will be needed to linearize the amplifier.

15-3.5 The circuit configuration is generally the same as that of Figure 15-11, with a pi-network used to provide the drain load resistance and gate bias supplied through an operational amplifier.



For a 4-Watt AM signal, P_{oPEP} = 20 W, hence

$$R = \frac{12^2}{2 \cdot 20} = 3.60\ \Omega$$

$$V_{cmPEP} = 12\ \text{V}$$

$$i_{D,max} = I_{cmPEP} = V_{cmPEP}/R = 3.33\ \text{A}$$

$$v_{D,max} = V_{DD} + V_{omPEP} = 24\ \text{V} \quad .$$

The worst-case dissipation rating should allow for accidental setting of the carrier level. This amplifier should be about as efficient than a Class B linear amplifier, hence by doubling (12-17) to include the dissipation in both transistors of a push-pull pair,

$$P_{d,max} \cong 2 \cdot \frac{12^2}{\pi^2 (3.6)} = 8.11\ \text{W.}$$

Since an operating frequency was not specified, component values for the output circuitry cannot be determined. However, $X_{RFC_1} >> 360$ ohms and $X_{C_2} << 0.036$ ohms, and the reactances of the components of the pi-network can be determined. From the procedure used in [26] of Chapter 13 with $Q = 5$,

$$X_{C_3} = R/Q = 0.72\ \Omega$$

$$X_{C_4} = R_o \sqrt{\frac{R/R_o}{(Q^2 + 1) - (R/R_o)}} = 2.63\ \Omega$$

$$X_{L_1} = \frac{QR + (RR_o/X_{C_4})}{Q^2 + 1} = 3.32\ \Omega \quad .$$

Since Class B operation is to occur at *PEP*,

$$I_{DD} = I_{cm} = 3.33\ \text{A} \quad ;$$

this requires the RF driving voltage to be

$$V_{gs,\text{RF}} = I_{DD}/g_m = 8.33\ \text{V.}$$

The carrier level is obtained by setting

$$V_{GS} - V_T = -\tfrac{1}{2} V_{gs,RF} \quad ,$$

hence

$$V_{GS} = -2.17 \text{ V} \quad .$$

For $m_a = 1$,

$$V_{gs,AF} = \tfrac{1}{2} V_{gs,RF} = 4.17 \text{ V}.$$

Capacitor C_1 should have a relatively low impedance at RF and a relatively high impedance at AF; inductor RFC_2 should have a relatively high impedance at RF and a relative low impedance at AF (in comparison to the gate impedance of the FET. The operational amplifier is assumed to have a high input impedance and unity voltage gain. The gate bias voltage should be obtained from a regulated supply, assumed here to be a 6.3 V sensor diode connected to a -12 V supply through R_3 = 12 kΩ(hence I_z = 10 mA). The carrier level is adjusted by variation of R_3, which might be a 5 kΩ to 10 kΩ potentiometer; this value is not critical as long as it does not load D_1. Values of R_1 and R_2 are chosen to allow proper operational amplifier.

From the discussion in the text,

$$\eta_{PEP} = 78.5\% \text{ (Class B)}$$

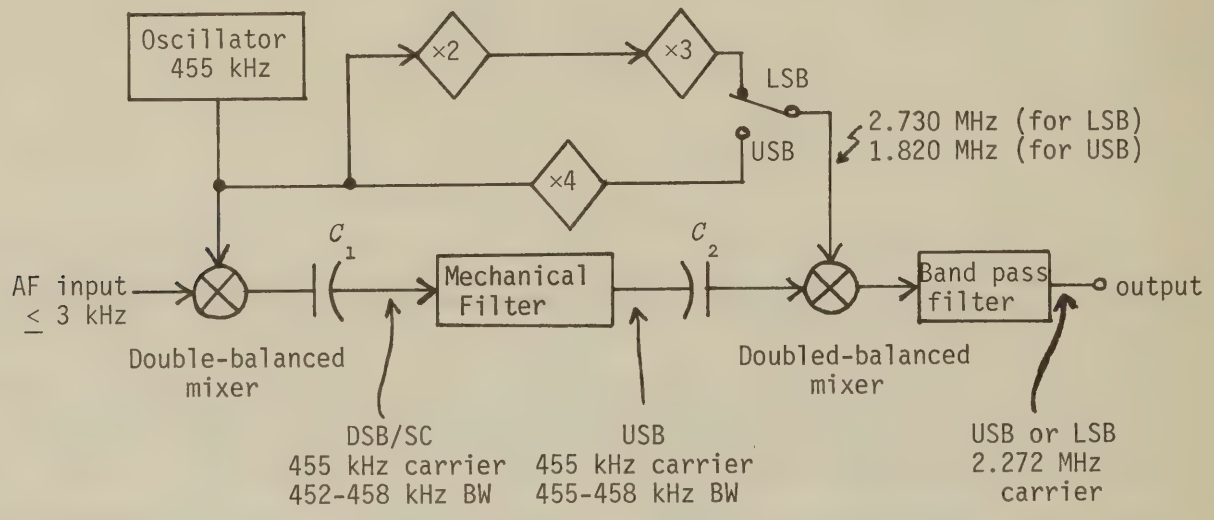
$$\eta_{CAR} \cong 39.3\% \text{ (no AF signal)}$$

and

$$\eta_{AVG} = \tfrac{3}{4} \eta_{PEP} = 52.4\% \ (m_a = 1, \text{ single tone}).$$

CHAPTER 16

16-1.1 When both upper- and lower-sideband SSB signals are to be produced with the same carrier frequency, the expenses of a second mechanical filter can be avoided by using selectable conversion frequencies that are harmonics of the basic oscillator frequency.



By selecting either the fourth or sixth harmonic of the 455 kHz oscillator, either LSB or USB signals with a (supressed) carrier frequency of 5·455 kHz = 2.272 MHz can be produced. Capacitors C_1 and C_2 are necessary with most mechnical filters to provide relatively high load impedances. Not shown is a switch to shut off the power to the unused set of frequency multipliers.

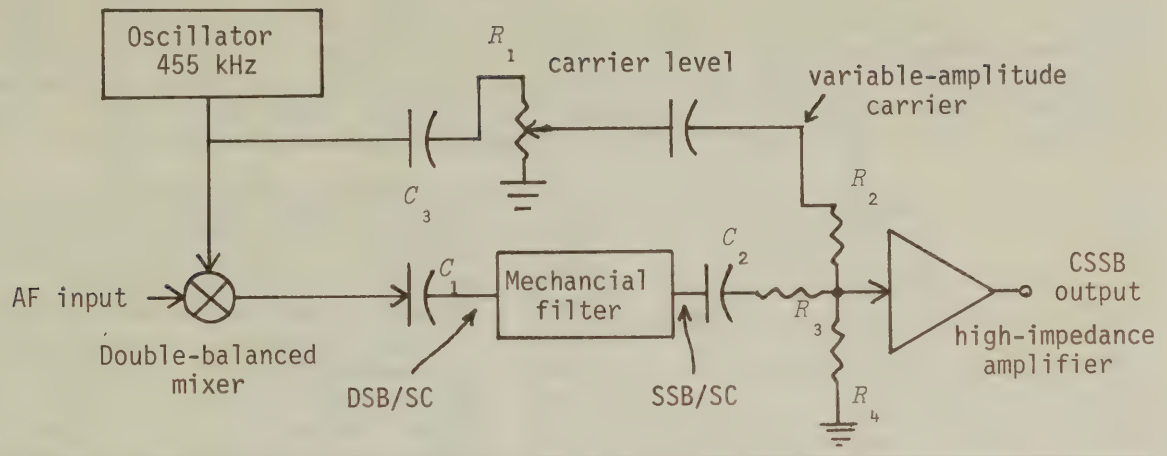
16-1.2 Compatible SSB is still a requirement for some military communications equipment, and reduced-carrier SSB can be used to allow syncronous detection, hence preservation of phase information and high fidelity. In theory, it would be possible to apply a full-carrier AM signal to a bandpass (mechanical) filter that passed one sideband and the carrier. However, the very sharp roll-off required makes such filters impractical. Therefore, CSSB and reduced-carrier SSB are usually generated by reinsertion of the carrier. On the next page, R_2, R_3, and R_4 form a summing network.

16-1.3 The frequencies 2.182 and 8.728 MHz are reserved for international-distress voice communications. To determine the block diagram for a distress-frequency transmitter, note the relationship between the two desired output frequencies:

$$f_{O2} = 8.728 \text{ MHz} = 4\, f_{O1} = 2.182 \text{ MHz} = 2 \cdot 1.091 \text{ MHz} = f_1,$$

where f_1 is a frequency that will be used internally. Now

Diagram for Problem 16-1.2

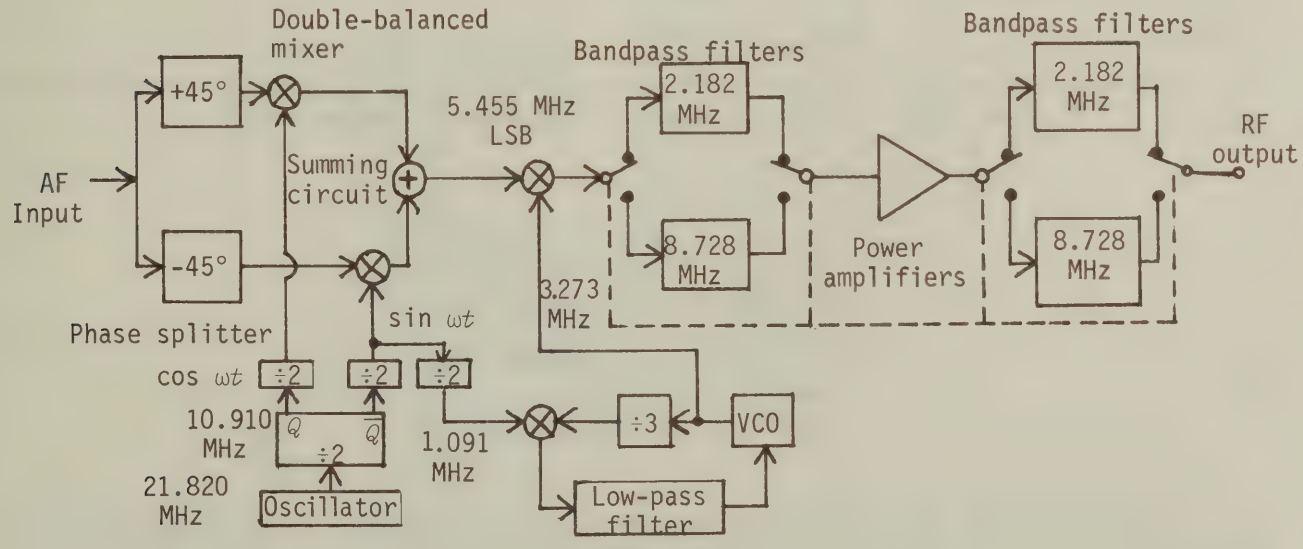


$$5 f_1 + 3 f_1 = 5.455 + 3.273 = 8.728 \text{ MHz} = f_{O2}$$

and

$$5 f_1 - 3 f_1 = 5.455 - 3.273 = 2.182 \text{ MHz} = f_{O1} \quad .$$

This suggests a mixing scheme in which all required frequencies are synthesized from a common oscillator. To obtain a lower-sideband signal on both output frequencies the 5.455 MHz frequency should be modulated. For simplicity and economy, a phasing-type modulator is suggested. The correct combination of AF and RF phase shifts for an LSB output can be determined by (1) Writing (8-9) with sine functions, (2) Converting it to a form analogous to (8-10), and (3) Adding the form just determined to (8-10) to see which sideband is cancelled.



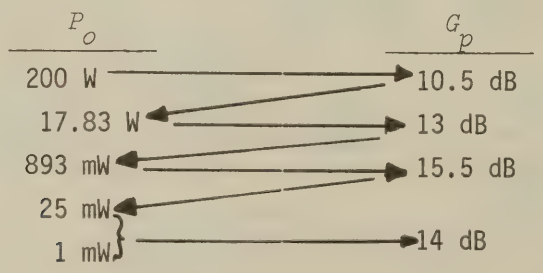
16-1.4 (a) The configuration for a discrete-band approach is similar to that of Figure 16-1. If the 8-8.5 MHz IF were directly translated to the 50-54 MHz range, stringent requirements would be placed on the bandpass filters to provide acceptable image rejection. Therefore a second oscillator, mixer, and filter were added to translate the 28-30 MHz signal to the 50-54 MHZ range. The modulator is addressed later. Seven sets of bandpass filters are required, but seventeen (17) subbands are required.

Band m	f_o MHz	f_1 MHz	f_2 MHz
160	1.6 - 2.1	10.1	off
80	3.5 - 4.0	12.0	off
40	7.0 - 7.5	15.5	off
20	14.0 - 14.5	22.5	off
15	21.0 - 21.5	29.5	off
10a	28.0 - 28.5	36.5	off
10b	28.5 - 29.0	37.0	off
10c	29.0 - 29.5	37.5	off
10d	29.5 - 30.0	38.0	off
6a	50.0 - 50.5	36.5	22.0
6b	50.5 - 51.0	37.0	22.0
6c	51.0 - 51.5	37.5	22.0
6d	51.5 - 52.0	38.0	22.0
6e	52.0 - 52.5	36.5	24.0
6f	52.5 - 53.0	37.0	24.0
6g	53.0 - 53.5	37.5	24.0
6h	53.5 - 54.0	38.0	24.0

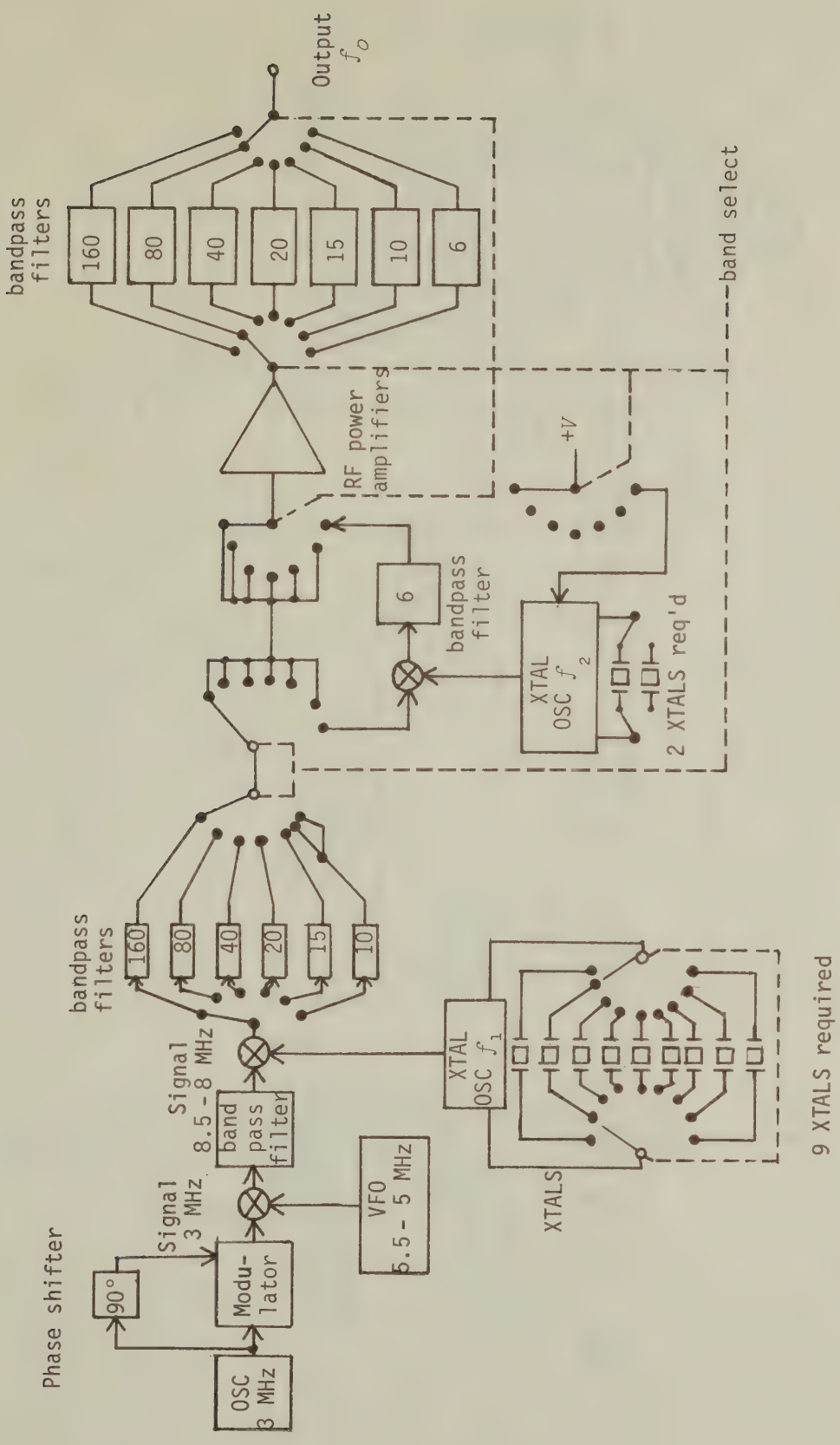
(b) The broadband configuration of the all-band amateur transmitter is similar to that of Figure 16-2 and a lot less complicated than the discrete-band approach. If the 1979 World Administrative Radio Conference allocates additional frequency ranges, the only changes needed are the addition of suitable output bandpass filters. The center frequency of the frequency synthesizer or variable-frequency oscillator is 142.8 MHz, hence the ±26.2 MHz tuning is only about 18 percent of the center frequency.

Modulator The modulator for this all-mode transmitter uses (1) the filter method of generating SSB, (2) direct electronic switching for CW, (3) reinsertion of an in-phase carrier for AM, and (4) insertion of a phase-quadrature carrier (Armstrong method) for FM.

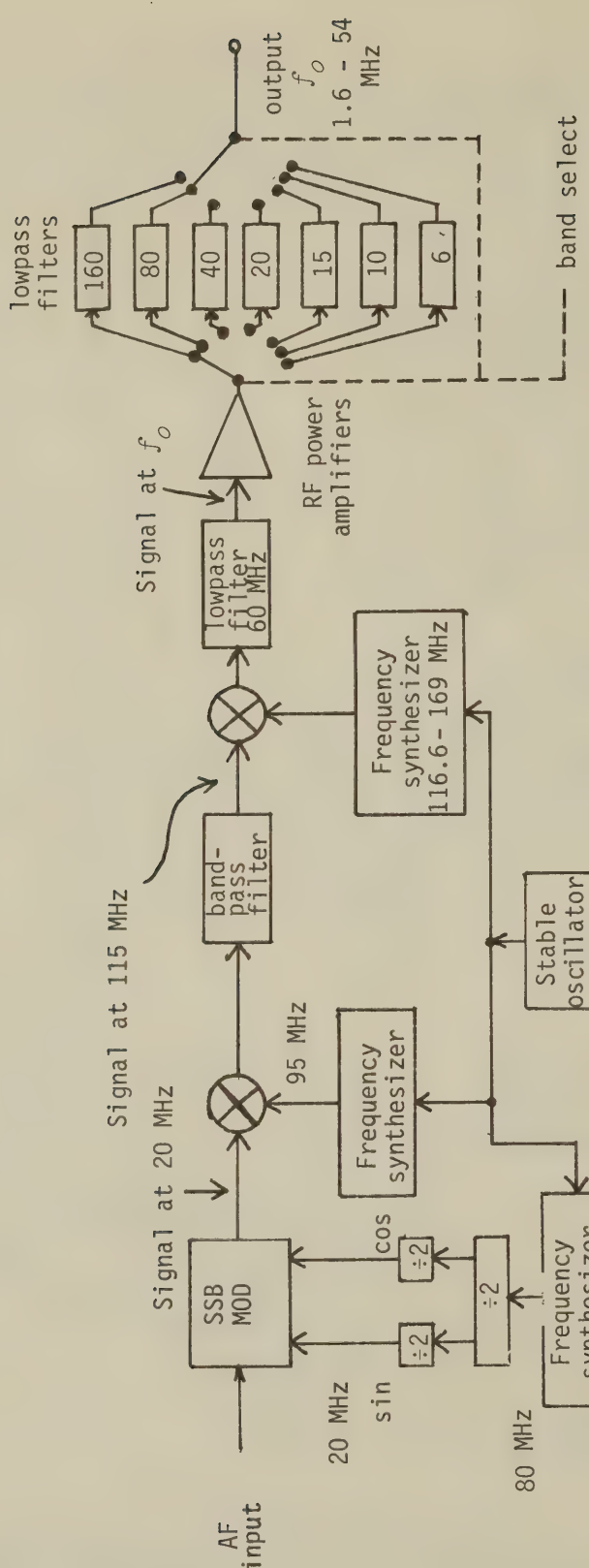
16-2.1 First, use Figure 15-1 to develop a rough gain-and-power chart:



Since the lowest-level stage can have about 18 dB gain, its gain is increased over the rough estimate and the gains of the higher-power stages are decreased accordingly, producing the formal gain-and-power chart: (see next page)

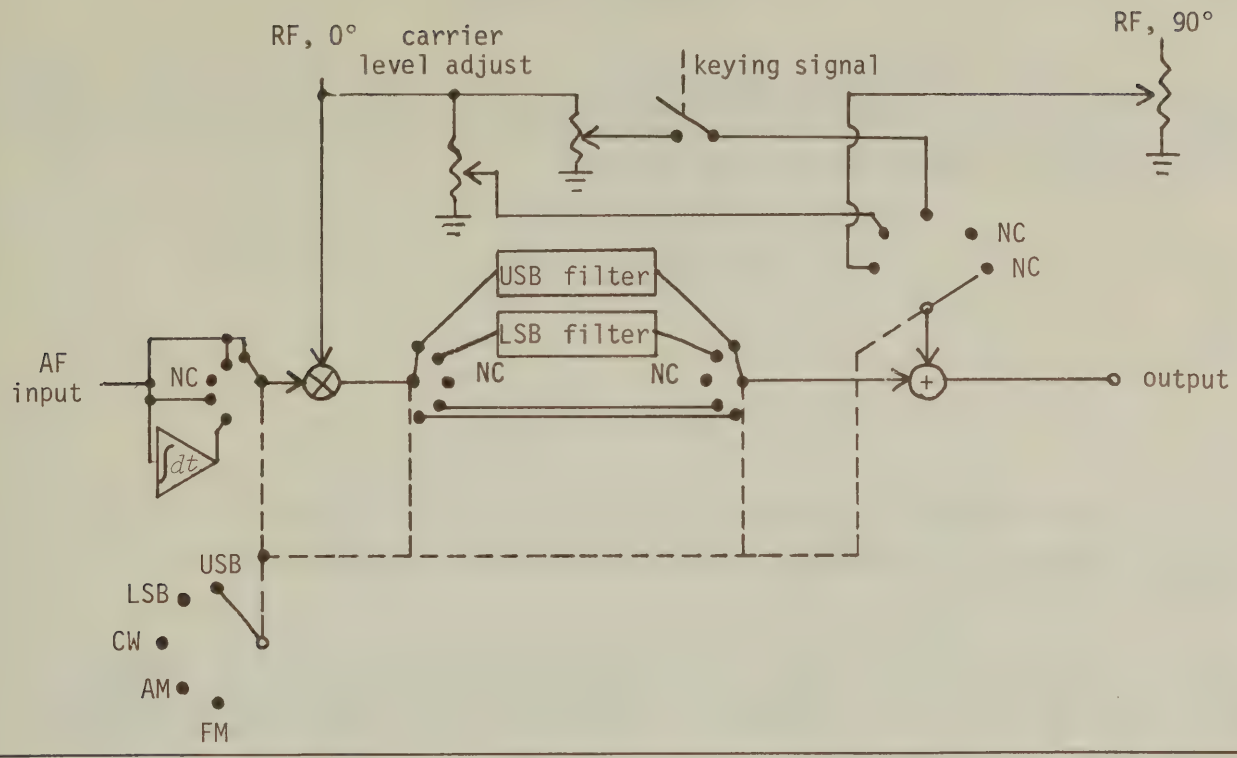


Problem 16-1.4(a)

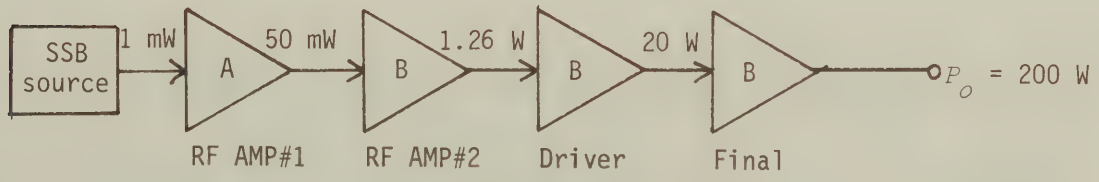


Probelm 16-1.4(b)

Problem 16-1.4 (modulator)



Stage	Class	P_o, W	G_p, dB	η	P_i, W
Source	-	0.001	-	-	-
RF AMP#1	A	0.050	17	0.3	0.17
RF AMP#2	B	1.26	14	0.55	2.29
Driver	B	20	12	0.55	36.36
Final	B	200	10	0.55	363.64



From the table,

$$P_{i,XMTR} = 402.46 \text{ W}$$

hence $\eta = 200/402.46 = 49.7\%.$

The first RF amplifier stage operates in Class A at a very low signal level; consequently, its contribution to the distortion in the transmitter output should be very small and can be ignored. (Such an assumption should, of course, be verified by additional analysis). Adaptation of (16-2) produces

$$3\, d_i \leqq \frac{1}{31.62} \sim -30 \text{ dB}$$

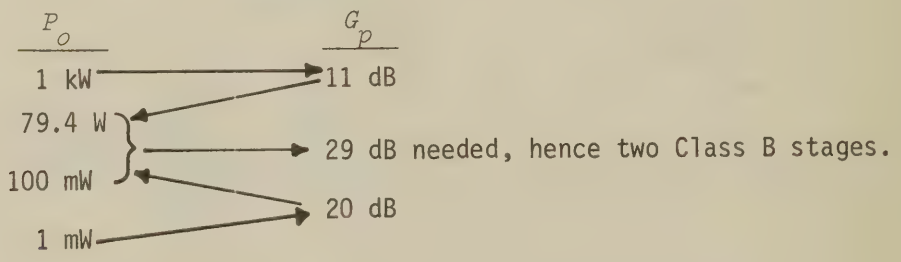
hence

$$d_i \leqq \frac{1}{94.87} \sim -39.5 \text{ dB},$$

where d_i is the distortion (voltage) ratio for any of the three Class B RF stages. Note that the same result is obtained by adding 20 log (1/3) ≅ -9.5 dB to the -30 dB transmitter *IMD* specification.

16-2.2 Four 250-Watt PAs must be combined to obtain the required 1-kW transmitter output. The power can be split and combined in two steps (using the spliters and combiners discussed in Section 12-7) or split and combined in a single step using the networks discussed in [27] of Chapter 12.

To design the chain, first rough out the gain-and-power chart by using the information given in the problem:



Now prepare the formal gain-and-power chart:

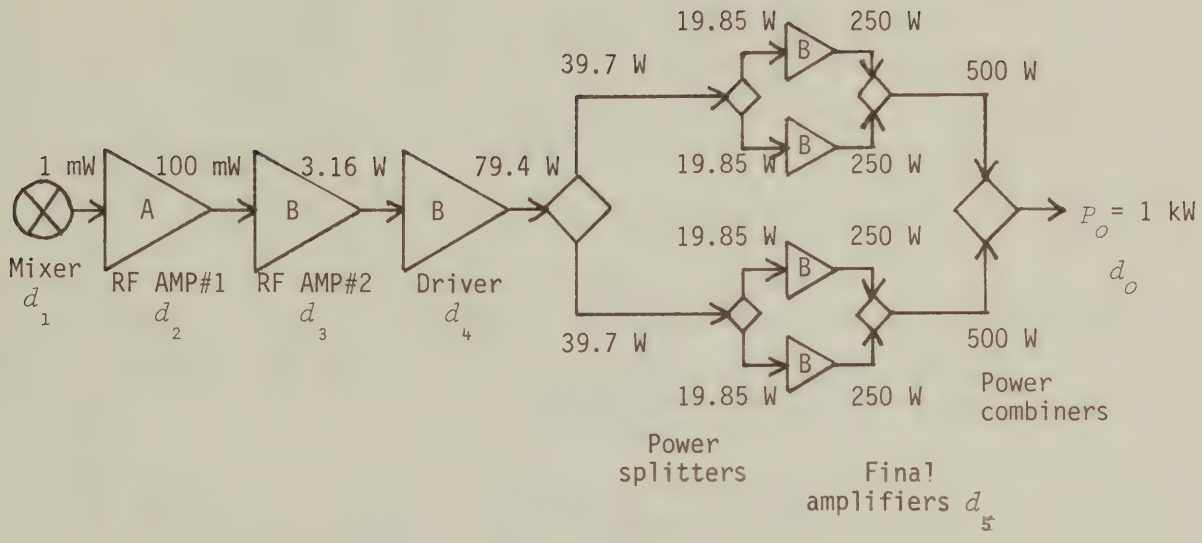
Stage	Class	P_o, W	G_p, dB	η	P_i, W
Mixer	-	0.001	-	-	-
RF AMP#1	A	0.100	20	0.3	0.33
RF AMP#2	B	3.16	15	0.55	5.7
Driver	B	79.4	14	0.55	144.4
Final	B	1000	11	0.55	1818.2

From the above,

$$P_{i,XMTR} = 1968.3 \text{ W}$$

$$\eta_{XMTR} = 1000/1968.3 = 50.8\%.$$

$$I_{dc} = \frac{P_{i,XMTR}}{V_{CC}} = \frac{1968.3}{28} = 70.3 \text{ A} .$$



Use an adaptation of (16.2) to determine the *IMD* requirements on each stage:

INPUT	-60 dB → d_1	= 0.00100
FINALS	-38 dB → d_5	= 0.01259
OUTPUT	-30 dB → d_o	= 0.03162

$$d_2 + d_3 + d_4 \leq d_o - d_1 - d_5 = 0.01803$$

If $d_2 = d_3 = d_4$, then

$$d_2 = d_3 = d_4 \leq \frac{0.01803}{3} = 0.00601 \sim -44.4 \text{ dB}.$$

16-3.1 Power consumption and dissipation ratings vary considerably with the type of signal. For example, many commercially-made amateur transmitters were quite reliable for SSB and CW use at the same *PEP*, but would burn up if used with RTTY or FM signals of the same *PEP*. To determine the performance of this two-stage transmitter, prepare a power output, efficiency, and power input table for each signal.

(a) FM signals have a 100% duty ratio, hence P_{oPEP} is used:

Stage	P_o, W	η	P_i, W
A	5	0.35	14.3
B	100	0.65	153.8

$$P_{i,XMTR} = 168.1 \text{ W}$$

$$\eta_{XMTR} = 100/168.1 = 59.5\% .$$

(b) CW signals have a 50% duty ratio, hence the power consumed is the average of that for key up and key down conditions.

Class	Key	P_o, W	P_i, W	P_{iAVG}
A	up	0	14.3	14.3
	down	5	14.3	
B	up	0	0	76.9
	down	100	153.8	

$$P_{iAVG,XMTR} = 91.2 \text{ W}$$

$$P_{oAVG} = \tfrac{1}{2}(100) = 50 \text{ W}$$

$$\eta_{XMTR} = 50/91.2 = 54.8\%.$$

(c) Equation (16-11) gives the average efficiency (0.617) of an ideal Class B

stage for two-tone SSB signals. This is multiplied by the *PEP* efficiency (0.65) of the real Class B PA to get the average efficiency of the real Class B PA:

$$\eta_{AVG} = 0.617 \cdot 0.65 = 0.401$$

Since $\xi = 2$ (~ 3 dB) for two-tone SSB,

$$P_{oAVG} = 100/2 = 50 \text{ W}$$

Class	P_{oAVG}, W	η	P_{iAVG}, W
A	2.5	-	14.3
B	50	0.401	124.7

$$P_{i,XMTR} = 139.0$$

$$\eta_{XMTR} = 50/139.0 = 36.0\% \quad .$$

(d) Equation (16-10) gives the average efficiency (0.589) of an ideal Class B PA, and $\xi = 8/3$. As in (c) above,

$$\eta_{AVG} = 0.589 \cdot 0.65 = 0.383$$

$$P_{oAVG} = 100/(8/3) = 37.5 \text{ W}.$$

Class	P_{oAVG}, W	η	P_{iAVG}, W
A	1.875	-	14.3
B	37.5	0.383	97.9

$$P_{i,XMTR} = 112.2 \text{ W}$$

$$\eta_{XMTR} = 37.5/112.2 = 33.4\%$$

(e) Equation (16-12) gives the approximate average efficiency of an ideal Class B PA with speech-derived SSB. Since 10 dB is analogous to a $\xi = 10$,

$$\eta_{AVG} = (0.65)\frac{1.048}{\sqrt{10}} = 0.215;$$

note that the $\pi/4$ efficiency of an ideal PA is replaced by the 0.65 efficiency of the real PA. Also,

$$P_{oAVG} = P_{oPEP}/\xi = 100/10 = 10 \text{ W}$$

Class	P_{oAVG}, W	η	P_{iAVG}, W
A	0.5	-	14.3
B	10	0.215	46.4

$$P_{i,XMTR} = 60.7 \text{ W}$$

$$\eta_{XMTR} = 10/60.7 = 16.5\% \quad .$$

16-3.2 Since the ten 1-Watt signals are of different frequencies, their powers add to produce

$$P_{oAVG} = 10\ (1) = 10 \text{ W} ,$$

for the repeater. However, their peak voltages add to produce the peak voltage output (hence peak envelope power) of the repeater, thus

$$P_{oPEP} = (10\sqrt{1})^2 = 100 \text{ W}.$$

$$\xi = P_{oPEP}/P_{oAVG} = 100/10 = 10 \quad .$$

For a Class A PA,

$$\eta_{AVG} = \frac{1}{2\xi} = \frac{1}{2 \cdot 10} = 5\% \quad .$$

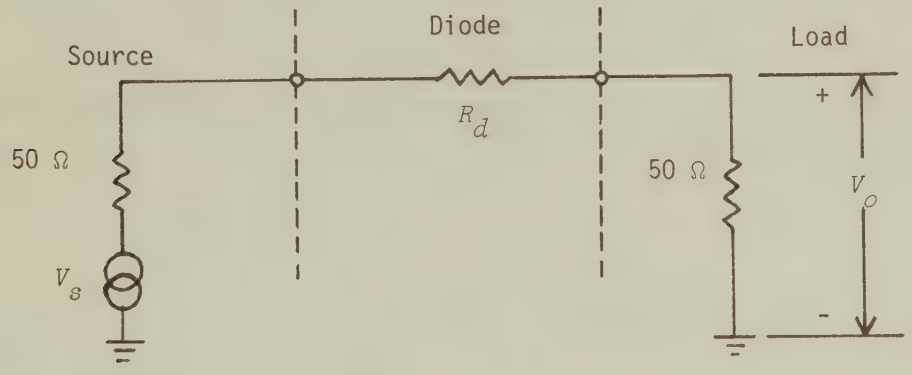
$$P_i = 2\ P_{oPEP} = 200 \text{ W}.$$

For a Class B PA, (16-12) can be applied by assuming a gaussian envelope, thus

$$\eta_{AVG} \cong \frac{\pi}{4}\frac{1.048}{\sqrt{10}} = 26.0\%$$

$$P_{iAVG} = P_{oAVG}/\eta_{AVG} = 38.4 \text{ W.}$$

16-4.1 The circuit diagram of Figure 16-6b applies and reduces to the equivalent circuit below for RF:



(The designer should remember that if the source and/or load are power amplifiers, they may not have the 50-ohm impedances shown.) Through ordinary circuit analysis, we obtain

$$V_o = \frac{50}{100 + R_d} V_s$$

or

$$\frac{V_o}{V_s} = \frac{1}{2 + R_d/50} ,$$

hence

$$-27.2 \text{ dB} \sim 0.044 \leq \frac{V_o}{V_s} \leq 0.424 \sim -7.5 \text{ dB} .$$

Since this system has $V_o/V_s = \frac{1}{2}$ with $R_d = 0$, the actual minimum insertion loss is 1.5 dB rather than 7.5 dB. The control range is 20 log (0.424/0.044) = 19.7 dB. Minimum attenuation is obtained with $I_{dc} = 1$ mA. Equation (16-13) is then used first to determine constant K and then I_{dc} for maximum attenuation:

$$K = 18 \cdot 0.001^{0.88} = 4.12 \cdot 10^{-2}$$

$$I_{dc,max\ atten} = \left(\frac{K}{1040}\right)^{1/0.88} = 9.95 \text{ μA}$$

The blocking capacitors should have $X_C \leq 18/10\ \Omega$ at $f = 2$ MHz, hence

$$C = 0.01\ \mu F + 100\ pF\ .$$

The RF chokes should have $X_{RFC} \geq (1040 + 50) \cdot 10 \cong 10\ k\ \Omega$, at $f = 2$ MHz, hence

$$L_{RFC} = 1\ mH\ .$$

16-4.2 Start with (16-14):

$$v_1(t) = v_o(t) - R_o\ i_o(t) \quad .$$

Convert this to a phasor relationship:

$$V_1 = V_o - R_o\ I_o$$

Now write V_o as a function of I_o and Z_o:

$$V_1 = (R + jX)\ I_o - R_o\ I_o$$

$$= [(R - R_o) + jX]\ I_o \quad .$$

The phasor magnitude is then

$$V_{1m} = |V_1| = \sqrt{[\mathrm{Re}(V_1)]^2 + [\mathrm{Im}(V_1)]^2}$$

$$= I_o \sqrt{(R - R_o)^2 + X^2} \quad .$$

16-4.3 A *VSWR* of $S = 2$ can be produced by an infinite number of load impendances. Since V_{1m} from the so-called *VSWR* detector is not actually *VSWR*, its value can differ from one impedance with $S = 2$ to another impedance with $S = 2$. The purpose of this problem is to find a trip point value of V_{1m} at which warning or drive reduction circuits are activated. To ensure that the alarm or protective circuitry is activated whenever $S \geq 2$, we must find the minimum value of V_{1m} for the set of load impedances with $S = 2$. This means, of course, that the protective circuitry will sometimes be activated with $S \leq 2$; however, this is preferable to the destruction of the transistors that might occur if $S \geq 2$

goes undetected.

The set of impedances with $S = 2$ is characterized by a given $|\Gamma|$ with $0 \leq \gamma < 2\pi$. To write V_{1m} as a function of γ, combine the formula for $|\Gamma|^2$ from Table 16-1 with (16-15):

$$|\Gamma|^2 = \frac{(V_{1m}/I_{om})^2}{(R + R_o)^2 + X^2}$$

$$V_{1m}^2 = I_{om}^2 \, |\Gamma|^2[(R + R_o)^2 + X^2] \quad .$$

Let

$$Z_1 = (R + R_o) + jX = R_o + Z$$

hence

$$Z_1^* = (R + R_o) - jX$$

and

$$V_{1m}^2 = I_{om}^2 \, |\Gamma|^2 (Z_1 \, Z_1^*) \quad .$$

From Table 16-1,

$$Z = \frac{1 + \Gamma}{1 - \Gamma} R_o \quad ,$$

therefore

$$Z_1 = R_o \left(\frac{1 + \Gamma}{1 - \Gamma} + 1 \right) = R_o \left(\frac{2}{1 - \Gamma} \right)$$

$$= R_o \left(\frac{2}{1 - \text{Re } \Gamma - j \text{ Im } \Gamma} \right)$$

$$Z_1^* = R_o \left(\frac{2}{1 - \text{Re } \Gamma + j \text{ Im } \Gamma} \right)$$

$$Z_1 \, Z_1^* = 4 \, R_o^2 \left[\frac{1}{(1 - \text{Re } \Gamma)^2 + (\text{Im } \Gamma)^2} \right]$$

Now

$$\text{Re } \Gamma = |\Gamma| \cos \gamma$$

$$\text{Im } \Gamma = |\Gamma| \sin \gamma$$

hence

$$Z_1 Z_1^* = \frac{4 R_o}{1 - 2 |\Gamma| \cos \gamma + |\Gamma|^2} \quad .$$

For a given I_{om} and $|\Gamma|$, V_{1m}^2 varies with $Z_1 Z_1^*$. The minimum values of V_{1m} therefore occurs with the minimum value of $Z_1 Z_1^*$ hence with

$$\cos \gamma = -1$$

and

$$\gamma = \pi \quad .$$

For $S = 2$,

$$|\Gamma| = \frac{S + 1}{S - 1} = \frac{1}{3} \, .$$

Since

$$e^{j\pi} = -1 \quad ,$$

$$\Gamma = |\Gamma| \, e^{-j\pi} = -\frac{1}{3} \quad ,$$

and

$$Z = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} R_o = \frac{2/3}{4/3} R_o = 25 \ \Omega \quad .$$

This gives

$$V_{1m} = I_{om} \sqrt{(25 - 50)^2 + 0^2} = 25 \, I_{om}$$

as the trip level for the protective circuitry.

Using similar reasoning, we find that the maximum value of V_{1m} for a given *VSWR* is produced when cos γ = +1, hence γ = 0 and Γ has a positive, real value. This *"VSWR"* detector can thus be tripped by a resistive load satisfying

$$(R - 50)^2 + 0^2 \geq 25^2 \quad ,$$

hence $R \geq 75\ \Omega$.

For this load,

$$S = \frac{75}{50} = 1.5 \quad .$$

We are therefore sure of a warning or drive reduction when $S \geq 2$, but may get warnings or drive reductions with S as low as 1.5.

16-5.1 The envelope and phase functions given by (16-17) and (16-18) are applicable to this problem, but the student must decide (a) whether to reference carrier phase to a sine or to a cosine wave and (b) which signs to use in $v_s(t)$ and $v_c(t)$ to produce an upper-sideband signal. Consider that

$$\sin \omega_m t \sin \omega_o t = \tfrac{1}{2}[\cos(\omega_o - \omega_m)t - \cos(\omega_o + \omega_m t)]$$

$$\cos \omega_m t \cos \omega_m t = \tfrac{1}{2}[\cos(\omega_o - \omega_m)t + \cos(\omega_o + \omega_m t)] \quad .$$

A USB signal is therefore produced by

$$v_o(t) = v_s(t) \sin \omega_m t + v_c(t) \cos \omega_m t \quad ,$$

where

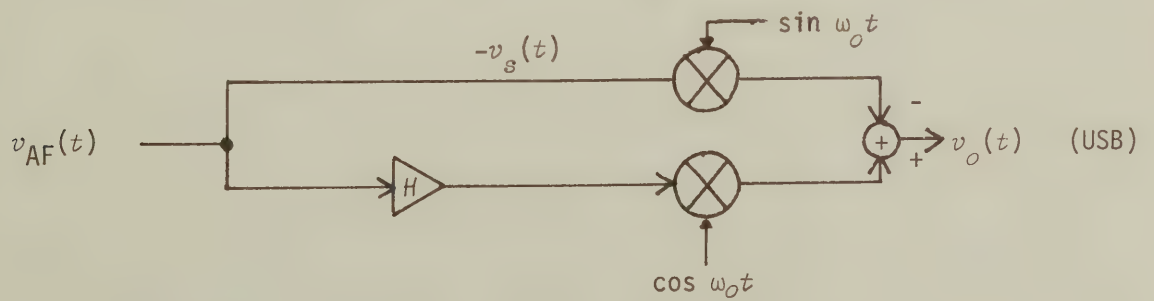
$$v_s(t) = -v_{AF}(t) = -\frac{1}{3}[\sin \omega_m t + \sin 3\omega_m t + \sin 5\omega_m t] \quad ,$$

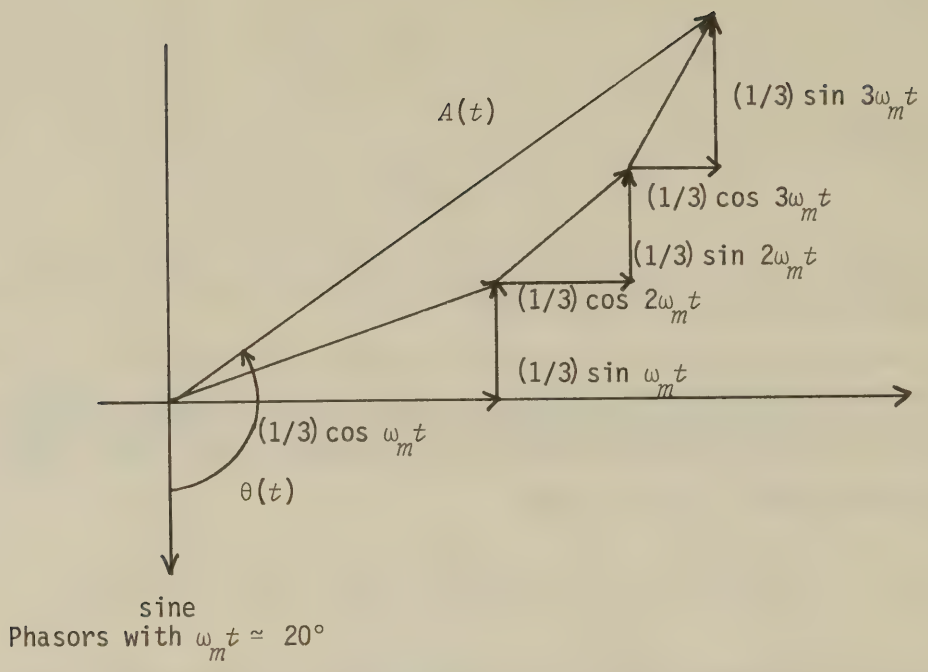
and

$$v_c(t) = H[v_{AF}(t)]$$

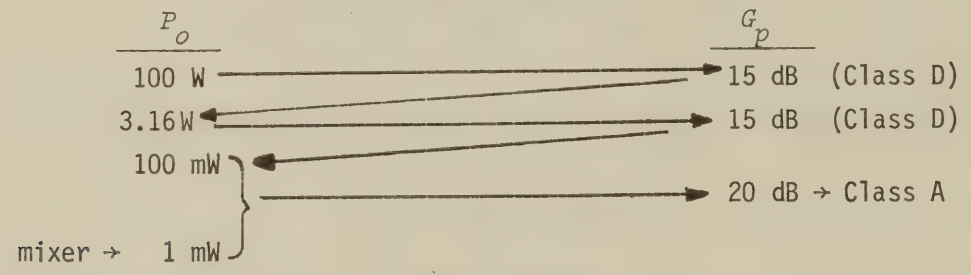
$$= \frac{1}{3}(\cos \omega_m t + \cos 3\omega_m t + \cos 5\omega_m t).$$

The phase function $\theta(t)$ is referenced to $\sin \omega_o t$. The 90° phase shift in $v_{AF}(t)$ is represented mathematically as a Hilbert transform ($H[\,]$).



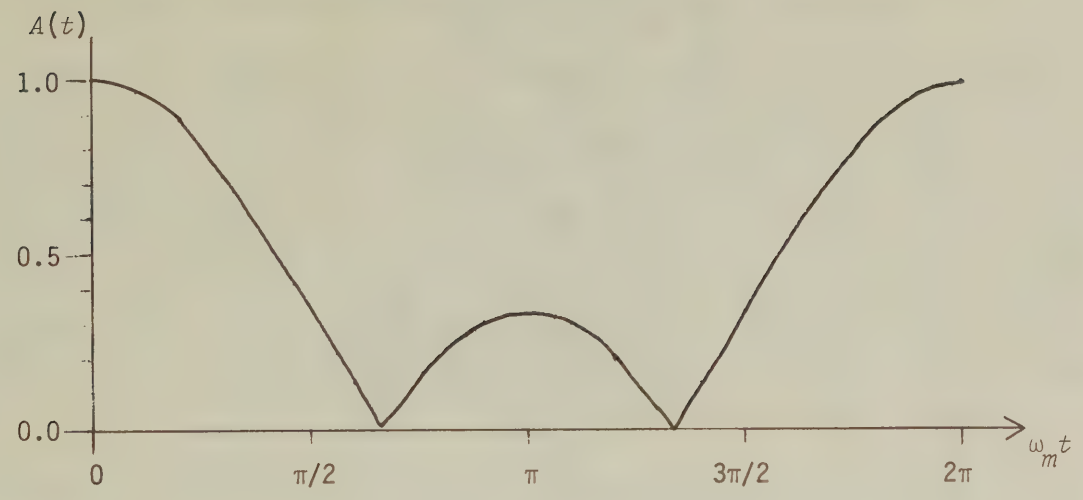


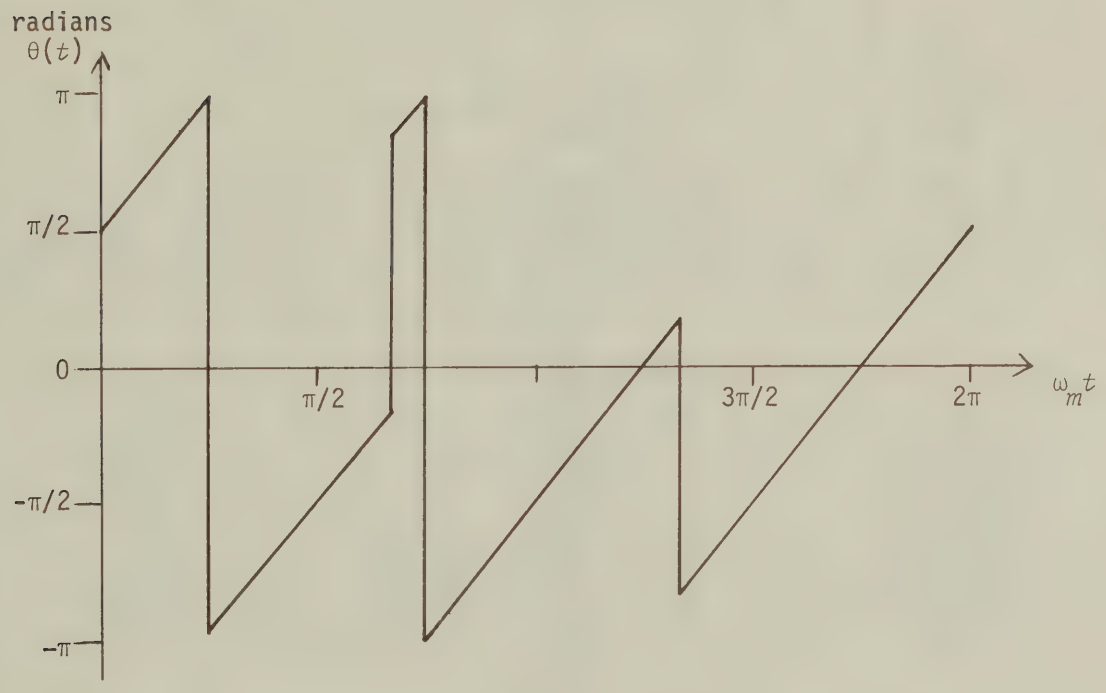
16-5.2 Designing an EER transmitter configuration is much the same as designing an AM transmitter configuration. First, rough out the RF power chain:

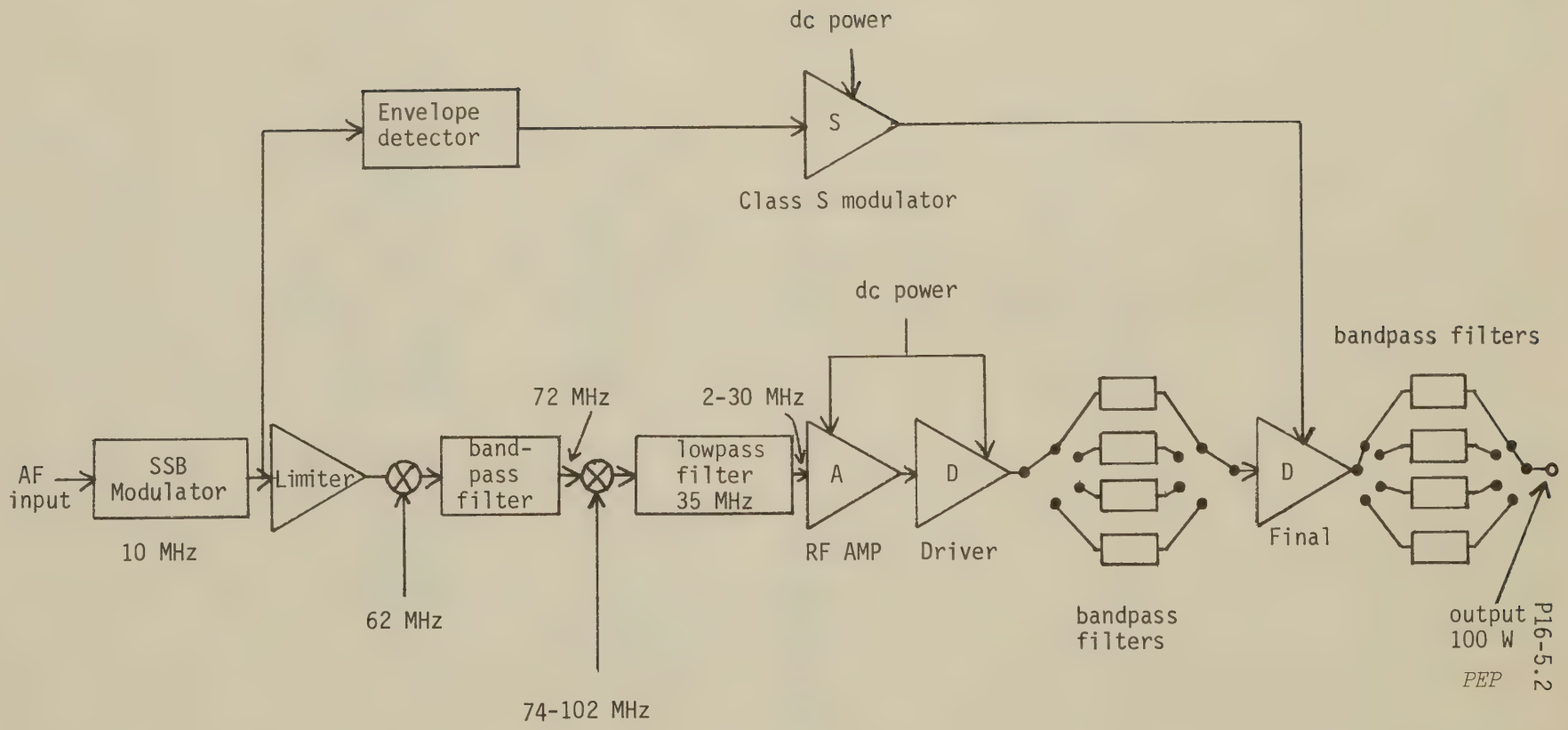


The broadband frequency translation scheme of Figure 16-2 is used. If fixed bandpass filters are used, at least 7 filters will probably be required to cover the 2 - 30 MHz range, as in Example 12-8.1; this is entirely suitable for the driver-to-final amplifier filtering. If the application allows, tunable output filters should be used to provide best performance and to cancel any reactance in the load. If tunable filters can be used, only four are necessary, as in Problem 14-2.3.

PROBLEM 16-5.1







PROBLEM 16-5.2

A *CW* signal of peak envelope power produces the following table:

Stage	Class	P_o, W	G_p, dB	η	P_i, W
Mixer	-	0.001	-	-	-
RF amp	A	0.100	20	0.35	0.29
Driver	D	3.16	15	0.8	3.95
Final	D	100.0	15	0.8	125.00
Modulator	S	125	-	o.85	147.06
Other	-	-	-	-	1.00

Since the final RF amplifier is powered by the output of the modulator,

$$R_{iXMTR} = 147.06 + 1.00 + 3.95 + 0.29 = 152.3 \text{ W}$$

$$\eta_{XMTR} = 100/152.3 = 65.7\%.$$

For speech-derived signals, the outputs and inputs of the Class D final RF amplifier and the Class S modulator must be reduced by the factor $1/\xi$. Since $\xi = 10$, the power consumed by the low-level stages becomes more significant. Nonetheless, the efficiency is higher than that of a transmitter using linear power amplification. (The Class B PA of Problem 16-3.1e consumed 46.4 W by itself.)

Stage	Class	P_{oAVG}, W	η	P_i, W
RF amp	A	0.100	0.35	0.29
Driver	D	3.16	0.80	3.95
Final	D	10.0	0.80	12.5
Modulator	S	12.5	0.85	14.71
Other	-	-	-	1.00

$$P_{iXMTR} = 14.71 + 1.00 + 3.95 + 0.29 = 19.95 \text{ W}$$

$$\eta_{XMTR} = 10/19.95 = 50.1\%.$$

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